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Verification of the Values of the Eigen Frequencies by the Modal Superposition Method

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Abstract: To ensure the certainty of results concerning Eigen frequencies values of a rectangular plate resting on the surface of an elastic half-space with inertial properties obtained numerically, verification is achieved by applying the principle of superposition method. To accomplish this operation, one must first determine the signal Tj(t) representing the sum of the amplitudes of the first Eigen modes at a point j of a plate. By applying the previously established signal formula of the modal method over a given time interval, the graph of the searched signal at a centre of any element of the discretized plate can be drawn. By studying the inverse problem of the obtained signal, through the application of the Fast Fourier Transform (FFT) via an appropriate computer code developed on Mathematica software, enabled spectrum location of the values of the Eigen frequencies of the plate in terms of the magnitude of the FFT. In this study, reading the obtained peaks of the spectrum allowed localization of the values of the first 16 Eigen frequencies of the studied plate. The natural frequency values of the studied plate were successfully verified by the principle of modal superposition.

Key words: Dynamic analysis / Eigen frequencies / Model method / Fourier transform FFT / Spectral peak

1. Introduction

The plates in interaction with elastic foundations considered like part of contact problems require deepening dynamical study to lead to a design and performances permitting its adequate application [1]. These types of plates are not completely studied to date given their complexity and always associated with a major mathematical difficulties [2]. In this context, a rectangular plate resting on the surface of the elastic half-space with inertial properties, Lamb's model [3], has been the subject of a dynamic study. A semi-analytical approach was used to perform this dynamic analysis involving the determination of the Eigen frequencies and Eigen modes of the plate, as well as its response due to external loads [4]. The values of the Eigen frequencies obtained are coming from the discretization of the plate [5], which automatically requires their verification using another approach. Therefore, the process of their verification is done by applying the principle of the modal superposition method [1,6].

2. Problem Aspect

The problem to solve is the verification of the values of the Eigen frequencies of a rectangular plate resting on the surface of an elastic half-space with inertial properties obtained numerically [5] by applying the modal method superposition [6,7,8].

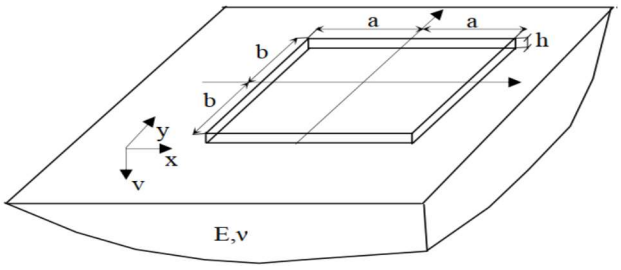

Fig. 1: Rectangular plate resting on the surface of an elastic half-space with inertial properties

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Therefore, one must determine the signal representing the sum of the amplitudes of the first sixteen Eigen modes of the plate at a given point j. The signal formula for this case is given by [6,7,8].

$$T_j(t) = \sum_{i=1}^{n=16} v_{ij} \cos\left(2\pi \omega_i t\right) \quad (1)$$

v_{ij} : Amplitude of the Eigen mode i of the plate at point j ;

ω_i : Eigen frequency i of the plate;

t : Modal response time;

$n = 16$: Number of the Eigen modes taken into account.

3. Problem Resolution

Application of formula (1) at a given time interval makes it possible to draw a graph of the searched signal at the center of any element of the discretized plate.

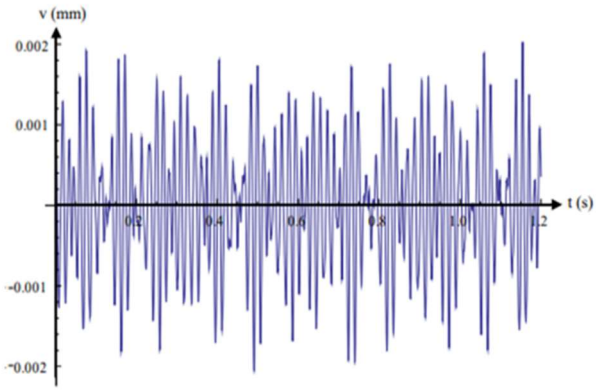


Fig. 2 Signal of the modal amplitudes v(mm) at a point of the plate given by the formula (1) versus time of vibration t(s)

Adding that the considered mesh is very fine, since the studied rectangular plate is divided into 900 identical rectangular elements [4].

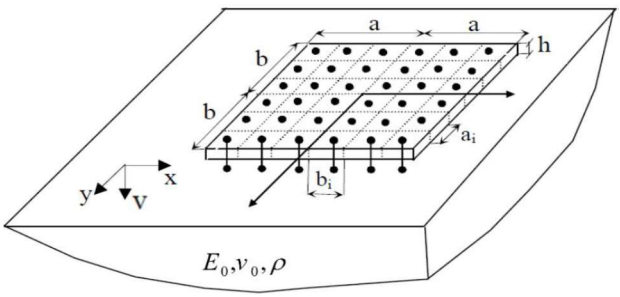


Fig. 3: Discretized plate

The study of the inverse problem of this signal by the application of the Fast Fourier Transform (FFT) via an appropriate computer code developed on Mathematica software enabled us to arrive at the spectrum locating the values of the Eigen frequencies of the plate, in terms of the FFT's magnitude, figure 4.

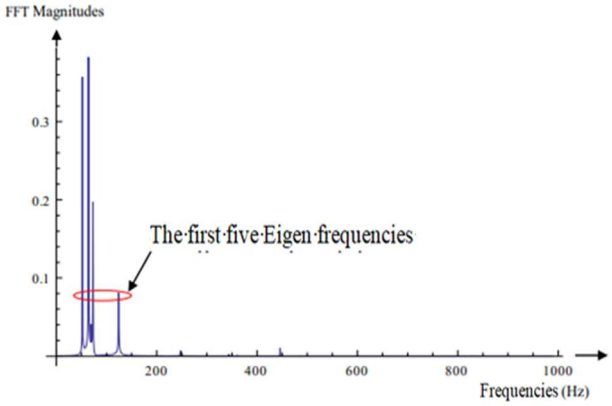
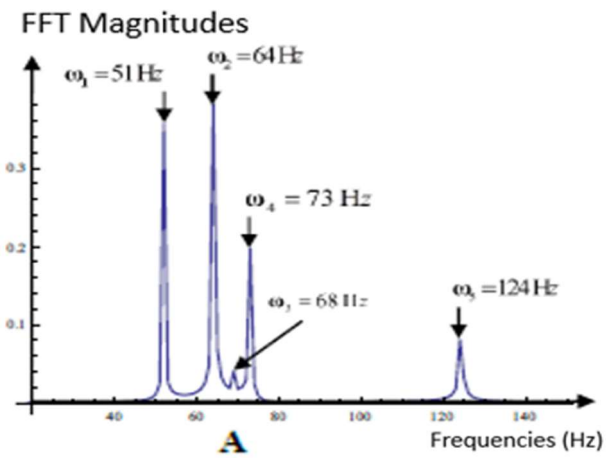


Fig. 4 Spectrum locating the first sixteen Eigen frequencies of the studied plate

Figure 4 represents the Fast Fourier Transform of the modal amplitudes signal. The reading of the obtained spectrum allows the localization of the first sixteen natural frequencies of the studied plate numerically. The peaks of the first Eigen frequencies of the plate are very clear, as shown in the fig. 4. Consequently, establishing their location is easy. The remainder of the peaks of the other Eigen frequencies do not appear on the spectrum of the fig. 4. Therefore, different frequency bands on the spectrum need to be examined, as shown in fig. 5.



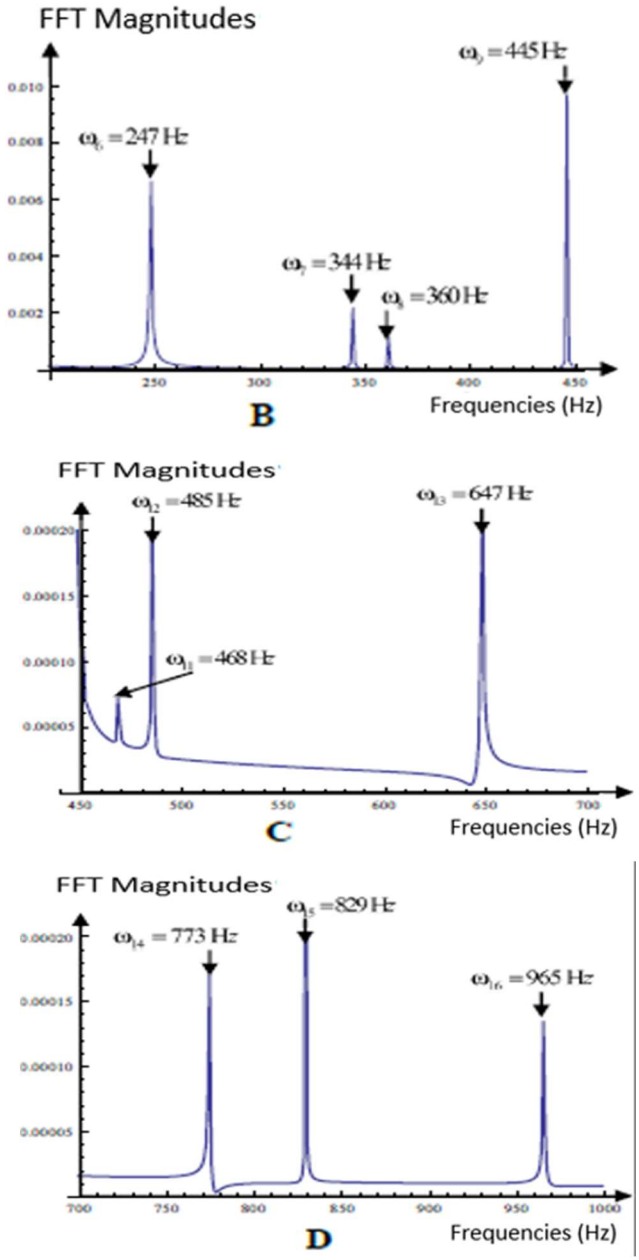


Fig. 5 Spectral zoom for the localization of the Eigen frequencies of the plate on different frequency bands

This zoomed in look of the spectrum on different frequency bands allows illustration of all the peaks of the first 16 Eigen frequencies of the plate and, consequently, the localization of their values.

4. Results and Discussion

Table 1 demonstrates a strong correspondence for the numerically obtained first eight Eigen frequencies generated by a semi-analytical approach based on the Zhemochkin method [5] to those readers on the spectrum.

Eigenfrequency number of the studied plate	Values obtained numerically (Hz)	Values obtained by the modal method (Hz)
ω_1	51.93084	51.93
ω_2	64.14663	64.15
ω_3	68.97462	68.97
ω_4	73.13283	73.13
ω_5	124.28131	124.28
ω_6	247.71888	247.72
ω_7	344.12066	344.12
ω_8	360.8515	360.85

Tab. 1 Comparison of the values of the Eigen frequencies obtained numerically with those obtained by the modal method

As is clearly shown the calculated values are the same ones located by the principle of superposition of the modal method.

Now remember how we obtained the values of Eigen frequencies of the studied plate numerically.

Starting from the canonical system of equations [5]:

$$\begin{cases} \vdots \\ \sum_{i=1}^{n}(v_{ij}+W_{ij})X_j - \sum_{i=1}^{n}W_{mi}J_i + \ell_{ix}\phi_{0x} + \ell_{iy}\phi_{0y} + u_0 = 0; \\ \vdots \\ \sum_{i=1}^{n}\left[X_j - J_j\right]\ell_{jx} = \mathrm{I}_{0x}\ddot{\phi}_{0x}; \\ \sum_{i=1}^{n}\left[X_j - J_j\right]\ell_{jy} = \mathrm{I}_{0y}\ddot{\phi}_{0y}; \\ \sum_{i=1}^{n}\left[X_j - J_j\right] = m\ddot{u}_0, \end{cases} \quad (2)$$

The two parameters of the previous system, which for we encountered a great challenge to overcome their resolution were: W_{ij} and v_{ij}. These two parameters W_{ij} and v_{ij} represent the deflections of the plate and

the vertical displacements of the surface of elastic half-space respectively.

The first challenge was overcome by using the energetical method of Ritz. After a series of complicated mathematical calculations led to the final formula permitting the determinations of the deflections of the studied plate (fig. 6).

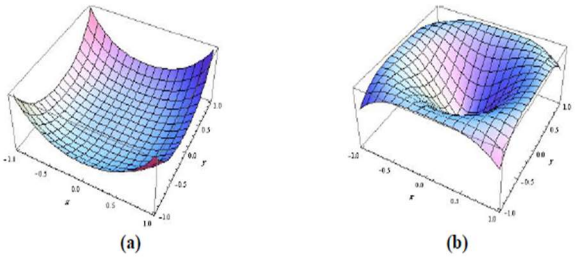


Fig. 6 Plate's deflections caused by: (a) uniformly distributed load over the entire surface of the plate; (b) concentrate force applied at the plate's centre

The second challenge was overcome by studding Green function using the complicated model Lamb model. The complexity of this model lies in taking into consideration the wave nature of the movements. This consideration is associated with major mathematical complexity which has resulted in the use of the special functions.

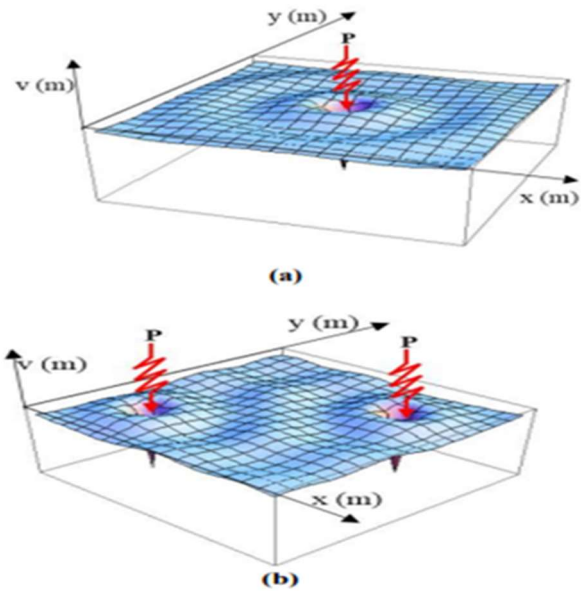


Fig. 7 Illustration of the displacement propagation of the elastic half-space surface at a given instant: (a) caused by one dynamic load; (b) caused by two dynamic loads

Figure 7 clearly shows the wave propagation from one external dynamic load and the affiliated displacements across the elastic half-space surface in all directions (a), as well as overlapping displacement waves resulting from distinct dynamic loads (b).

5. Results Comparison

The results were compared to Boussinesq's model, where there is no wave phenomenon (fig. 8).

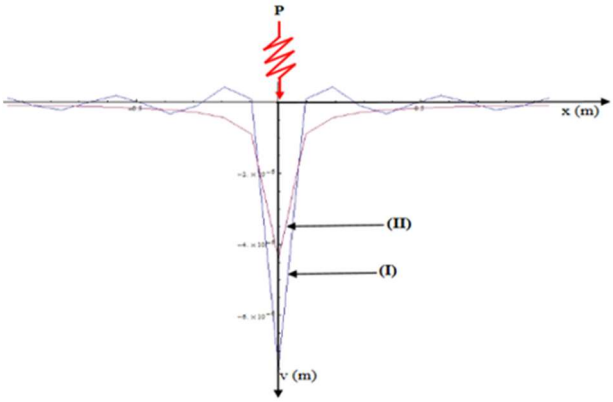


Fig. 8 Form of the vertical displacements of the elastic half-space surface due to a dynamic load: (I) Lamb's model with wave phenomenon; (II) Boussinesq's model without wave phenomenon

Comparison of the values of the first six natural frequencies of the studied plate resting on the surface of an elastic half-space considering two models is given in table 2.

Eigen Frequency	$\omega 1$	$\omega 2$	$\omega 3$	$\omega 4$	$\omega 5$	$\omega 6$
Lamb Model	51,93084	64,14663	68,97462	73,13283	124,28131	247,71888
Boussinesq Model	52,00765	64,17299	69,03797	73,17534	124,31682	247,73490

Tab. 2 Comparison of the values of the Eigen frequencies obtained by Lamb model and Boussinesq model

This demonstrates the advantage of the studied model, which is closer to reality than other simplifying models, which in the case of Boussinesq under predicts.

6. Conclusions

Verification of the Eigen frequencies values of the studied plate obtained numerically is indispensable by

any means. We chose the principle of the modal method superposition to confirm the reliability of the approach. The values of the frequencies shown as peaks on the FFT graphs correspond exactly to the values of the Eigen frequencies of the studied plate obtained numerically by the proposed approach. This confirms the reliability of the approach and consequently the certainty of its results.

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