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Equilibrium evaluation of pressure-strain correlation models for compressible homogeneous sheared turbulence

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Abstract: Rapid distortion theory (RDT) is an important tool for modeling turbulent flows. This theory is an analysis of linear stability allowing us to predict behavior of turbulent field in presence of mean strain and in the absence of inertial effects. The aim of this paper is to simulate homogeneous compressible sheared turbulence using RDT. Numerical simulations are carried out with an RDT validated code resolving unsteady linearized equations governing double correlations spectra evolution. These simulations will be sufficient to be used for evaluation of equilibrium states of the compressible models of Fujiwara et al. and Huang et al. concerning pressure-strain correlation. A certain concordance between RDT and models results is observed.

Keywords: Numerical simulation, turbulence, homogeneous, compressible, rapid distortion theory (RDT).

1. Introduction

Many researchers are interested in the use of a linear approach to modeling compressible homogenous sheared turbulence: the rapid distortion theory (RDT) (Cambon et al. [1], Simone et al. [2], Riahi et al. [3]). This theory permits us to well identify compressibility effects on structure of shear flows and increase our comprehension of turbulence phenomena. Riahi et al. [3] showed that this theory can be used to study equilibrium states of homogeneous sheared turbulence. So, with RDT, we will analyze the performances of the compressible models of Fujiwara et al. [4] and Huang et al. [5] concerning pressure-strain correlation. The evaluation of these models is carried out using results obtained by an RDT code developed and validated by Riahi et al. [3].

In Section 2, equations describing the compressible homogeneous sheared turbulence in the spectral space are presented as well as numerical method used to solve unsteady linearized equations governing double correlations spectra evolution. The third part is devoted to the presentation of compressible models to be tested. Results of comparison between RDT and models of Fujiwara et al. [4] and Huang et al. [5] are given in section 4. Finally, we conclude this work.

2. Mathematical modeling

2.1 Equations in the spectral space

The flow to be considered was a compressible homogeneous turbulent shear flow. We retained the same RDT equations adopted by Simone [6], Simone et al. [2]. The linearized equations of continuity and momentum controlling the fluctuating of velocity u_i and pressure p led to the following equations:

$$\dot{u}_i + u_j \frac{\partial \overline{U}_i}{\partial x_j} = -\frac{1}{\overline{\rho}}\frac{\partial p}{\partial x_i} + \frac{\lambda+\mu}{\overline{\rho}}\frac{\partial^2 u_j}{\partial x_i \partial x_j} + \frac{\mu}{\overline{\rho}}\frac{\partial^2 u_i}{\partial x_j^2}, \quad (1)$$

$$\frac{\dot{p}}{\gamma \overline{p}} = -u_{i,i}. \quad (2)$$

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The Fourier transform of equations (1) and (2) gave equivalent equations in the spectral space:

$$\dot{\hat{u}}_i + \frac{\mu}{\overline{\rho}} k^2 \hat{u}_i + \lambda_{ij}\hat{u}_j + \frac{\lambda+\mu}{\overline{\rho}} k_i k_j \hat{u}_j = -I k_i \frac{\hat{p}}{\overline{\rho}}, \quad (3)$$

$$\left(\frac{\dot{\hat{p}}}{\gamma \overline{P}}\right) = -\hat{u}_{i,i}. \quad (4)$$

Where $\lambda_{ij} = S\delta_{i1}\delta_{j2}$ is the mean velocity gradient and $I^2 = -1$.

The Fourier transform of the velocity field can be written, in the local reference of Craya-Herring, as

$$\hat{u}_i(\vec{k},t) = \hat{\varphi}^1(\vec{k},t)e_i^1(\vec{k}) + \hat{\varphi}^2(\vec{k},t)e_i^2(\vec{k}) + \hat{\varphi}^3(\vec{k},t)e_i^3(\vec{k}), \quad (5)$$

where $\hat{\varphi}^1(\vec{k},t)$ and $\hat{\varphi}^2(\vec{k},t)$ are the solenoidal modes and $\hat{\varphi}^3(\vec{k},t)$ is the dilatational mode.

2.2 Application: Case of the pure plane-shear

Based on the local reference of Craya-Herring decomposition, we retained the following equations system:

$$\dot{\hat{\varphi}}^1 + \upsilon k^2 \hat{\varphi}^1 + \frac{Sk_3}{k}\hat{\varphi}^2 + \frac{Sk_2k_3}{kk'}\hat{\varphi}^3 = 0, \quad (6)$$

$$\dot{\hat{\varphi}}^2 + \left(\upsilon k^2 + \frac{Sk_1k_2}{k^2}\right)\hat{\varphi}^2 + \frac{Sk_1}{k'}\hat{\varphi}^3 = 0, \quad (7)$$

$$\dot{\hat{\varphi}}^3 - 2\frac{Sk_1k'}{k_2}\hat{\varphi}^2 + \left(\frac{4}{3}\upsilon k^2 + \frac{Sk_1k_2}{k^2}\right)\hat{\varphi}^3 + ak\hat{\varphi}^4 = 0, \quad (8)$$

$$\dot{\hat{\varphi}}^4 - ak\hat{\varphi}^3 = 0. \quad (9)$$

Where k_1, k_2 and k_3 are the components of the wave vector $\vec{k}$ and $k' = \sqrt{k_1^2 + k_3^2}$. Here S denoted the shear rate ($S = \frac{d\overline{U}_1}{dx_2}$ = constant).

2.3 Doubles correlations

The spectral tensor of the doubles correlations can be expressed as

$$\Phi_{ij}(\vec{k},t) = \frac{1}{2}\left[\hat{\varphi}^{i*}(\vec{k},t)\hat{\varphi}^j(\vec{k},t) + \hat{\varphi}^i(\vec{k},t)\hat{\varphi}^{j*}(\vec{k},t)\right] \quad (10)$$

The asterisk is a complex conjugate.

Then, we wrote evolution equations of these doubles correlations (Riahi et al. [1,7]):

$$\frac{d}{dt}\Phi_{11} = -2\upsilon k^2\Phi_{11} - 2\frac{Sk_3}{k}\Phi_{12} + 2\frac{Sk_2k_3}{kk'}\Phi_{13}, \quad (11)$$

$$\frac{d}{dt}\Phi_{12} = \left(\frac{Sk_1k_2}{k^2} - 2\upsilon k^2\right)\Phi_{12} - \frac{Sk_1}{k'}\Phi_{13} - \frac{Sk_3}{k}\Phi_{22} + \frac{Sk_2k_3}{kk'}\Phi_{23}, \quad (12)$$

$$\frac{d}{dt}\Phi_{13} = 2S\frac{k_1k'}{k^2}\Phi_{12} - \left(\frac{7}{3}\upsilon k^2 + S\frac{k_1k_2}{k^2}\right)\Phi_{13} - ak\Phi_{14} - S\frac{k_3}{k}\Phi_{23} + S\frac{k_2k_3}{kk'}\Phi_{33}, \quad (13)$$

$$\frac{d}{dt}\Phi_{14} = ak\Phi_{13} - \upsilon k^2\Phi_{14} - \frac{Sk_3}{k}\Phi_{24} + \frac{Sk_2k_3}{kk'}\Phi_{34}, \quad (14)$$

$$\frac{d}{dt}\Phi_{22} = \left(-2\upsilon k^2 + 2\frac{Sk_1k_2}{k^2}\right)\Phi_{22} - 2\frac{Sk_1}{k'}\Phi_{23}, \quad (15)$$

$$\frac{d}{dt}\Phi_{23} = 2\frac{Sk_1k'}{k^2}\Phi_{22} - \frac{7}{3}\upsilon k^2\Phi_{23} - ak\Phi_{24} - \frac{Sk_1}{k'}\Phi_{33}, \quad (16)$$

$$\frac{d}{dt}\Phi_{24} = ak\Phi_{23} + \left(\frac{Sk_1k_2}{k^2} - \upsilon k^2\right)\Phi_{24} - \frac{Sk_1}{k'}\Phi_{34}, \quad (17)$$

$$\frac{d}{dt}\Phi_{33} = 4\frac{Sk_1k'}{k^2}\Phi_{23} - 2\left(\frac{4}{3}\upsilon k^2 + \frac{Sk_1k_2}{k^2}\right)\Phi_{33} - 2ak\Phi_{34}, \quad (18)$$

$$\frac{d}{dt}\Phi_{34} = 2\frac{Sk_1k'}{k^2}\Phi_{24} + ak\Phi_{33} - \left(\frac{4}{3}\upsilon k^2 + \frac{Sk_1k_2}{k^2}\right)\Phi_{34} - ak\Phi_{44}, \quad (19)$$

$$\frac{d}{dt}\Phi_{44} = 2ak\Phi_{34}. \quad (20)$$

Numerical integration of these equations was carried out using a simple second-order accurate scheme:

$$f(t+\Delta t) = f(t) + \Delta t f'(t) + \frac{\Delta t^2}{2} f''(t). \quad (21)$$

In the last equation, the derivatives $f'(t)$ and $f''(t)$ are expressed exactly from evolution equations (11) - (20) and Δt is the time-step size.

3. Presentation of compressible models

3.1 Spectral writing of double correlations

The pressure-strain correlations take the following forms:

$$\begin{aligned}\hat{\Pi}_{ij}\left(\vec{k},t\right) &= a\Phi_{41}\left(\vec{k},t\right)\left(k_j e_i^1\left(\vec{k},t\right)+k_i e_j^1\left(\vec{k},t\right)\right)\\ &+ a\Phi_{42}\left(\vec{k},t\right)\left(k_j e_i^2\left(\vec{k},t\right)+k_i e_j^2\left(\vec{k},t\right)\right)\\ &+ a\Phi_{43}\left(\vec{k},t\right)\left(k_j e_i^3\left(\vec{k},t\right)+k_i e_j^3\left(\vec{k},t\right)\right).\end{aligned} \quad (22)$$

3.2 Presentation of compressible pressure-strain correlation models

Model of Fujiwara et al.

These authors [4] proposed a model for pressure-strain correlation. The terms Π_{ij} as a function of the turbulent Mach number M_t and are written in the following form:

$$\Pi_{ij} = -f(M_t)\left[1.8\bar{\rho}\varepsilon b_{ij} + 0.6\left(P_{ij} - \frac{1}{3}P_{kk}\delta_{ij}\right)\right], \quad (23)$$

where $f(M_t) = 1 - exp\left(\frac{-0.02}{M_t^2}\right)$ (24)

and $P_{ij} = -\bar{\rho}\left(\overline{u_i u_m}\frac{\partial U_j}{\partial x_m} + \overline{u_j u_m}\frac{\partial U_i}{\partial x_m}\right).$ (25)

Model of Huang et al.

Huang et al. [5] also proposed a model for pressure-strain correlation and which is expressed as follows:

$$\begin{aligned}\Pi_{ij} &= -1.8\varepsilon b_{ij} + K\left[\frac{4}{5}\left(\tilde{S}_{ij} - \frac{1}{3}\tilde{S}_{kk}\right)\right.\\ &+ \left(0.6 + F(M_t)\right)\left(b_{ip}\tilde{S}_{pj} + b_{jp}\tilde{S}_{pi} - \frac{2}{3}b_{pq}\tilde{S}_{qp}\right)\\ &\left.+ \left(0.6 - F(M_t)\right)\left(\tilde{\omega}_{ip}b_{pj} - b_{ip}\tilde{\omega}_{pj}\right)\right],\end{aligned} \quad (26)$$

where $F(M_t) = 0.25exp\left(\frac{-0.05}{M_t^3}\right)$, (27)

$$\tilde{S}_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \quad (28)$$

and

$$\tilde{\omega}_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right). \quad (29)$$

4. Results and discussion

In figures 1 and 2, we presented evolution of components Π_{11}, Π_{12} and Π_{22} of the pressure-strain correlation models of Fujiwara et al. [4] and of Huang et al. [5]. Table 1 shows that results are given for two values of initial turbulent Mach number ($M_{t_0} = 0.3$ and $M_{t_0} = 0.5$) and for the same value of initial gradient Mach number ($M_{g_0} = 100$) corresponding to the compressible regime. In the asymptotic states, the Fujiwara et al. [4] model are in good agreement with RDT for Π_{11}, Π_{12} and Π_{22}. Moreover, these asymptotic states are independent of initial turbulent Mach number. On the other hand, Huang et al. [5] model gives results far from the forecasts of RDT with the exception of Π_{22} component which tends towards a zero value at equilibrium. We note that the different components Π_{11}, Π_{12} and Π_{22} tend towards a zero value at equilibrium for infinite non-dimensional times *St*. This value is independent of the initial turbulent Mach number. In the equilibrium states, it's clearly shown that there is no energy distribution between the different components b_{ij} of the Reynolds anisotropy tensor when $\left(\Pi_{ij}\right)_\infty = 0$.

Table 1 Initial conditions

Case	M_{g_0}	M_{t_0}
(A_1)	100	0.3
(A_2)	100	0.5

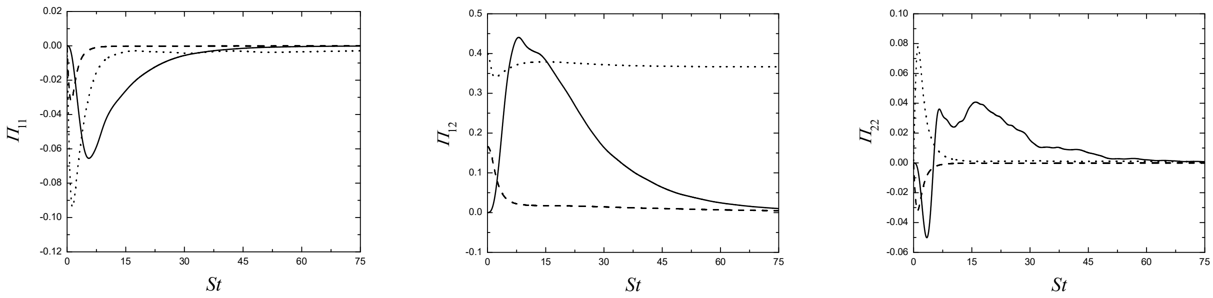

Fig. 1 Evolution of components Π_{ij} of the pressure-strain correlation in case (A_1). Solid line indicates RDT results, dashed line indicates Fujiwara et al. model, and dotted line indicates Huang et al. model.

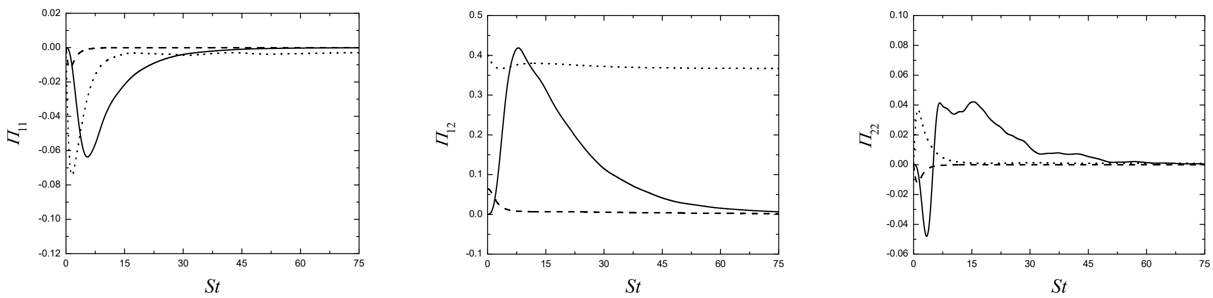

Fig. 2 Evolution of components Π_{ij} of the pressure-strain correlation in case (A_2). Symbols as in figure 1.

5. Conclusion

Rapid distortion theory (RDT) is used to evaluate some compressible models for homogeneous sheared turbulence. In this work, we have tested pressure-strain correlation models in the equilibrium states. The model of Fujiwara et al. [4] for components Π_{11}, Π_{12} and Π_{22} gives results that are close to zero but do not show any significant deviations from RDT. These results are independent of the initial turbulent Mach number. On the other hand, the Huang et al. [5] model gives results far from RDT except for the term Π_{22}. Obviously, the zero value of the different terms Π_{11}, Π_{12} and Π_{22} in equilibrium states can be interpreted by the not distribution of energy between the different components of the Reynolds anisotropy tensor.

Nomenclature

δ : Dirac delta

M_{g_0} : initial gradient Mach number

M_t : turbulent Mach number

M_{t_0} : initial turbulent Mach number

S : shear rate

St : non-dimensional times

γ : ratio of specific heats

u_i : velocity fluctuation

$\overline{U}$: mean velocity

p : pressure fluctuation

$\overline{P}$: mean pressure

$\overline{\rho}$: mean density

λ : second viscosity coefficient

μ : dynamic viscosity

υ : kinematic viscosity

λ_{ij} : mean velocity gradient

a : mean sound speed

Δt : time-step size

K : turbulent kinetic energy

b_{ij} : anisotropy tensor of Reynolds

ε : total dissipation rate of turbulent kinetic energy

References

[1] C. Cambon, G. N. Colemen, N. N. Mansour, Rapid distortion analysis and direct simulation of compressible homogeneous turbulence at finite Mach number, J. Fluid Mech. 257, pp. 641-665 (1993).

[2] A. Simone, G.N. Coleman, C. Cambon, The effect of compressibility on turbulent shear flow: a rapid-distortion-theory and direct-numerical-simulation study. J. Fluid Mech. 330, pp. 307-338 (1997).

[3] M. Riahi, T. Lili, Equilibrium states of homogeneous sheared compressible turbulence, AIP Advances. 1, Issue: 2, pp. 022117-022117-11, ISSN 2158-3226 (2011).

[4] H. Fujiwara, Y. Matsuo, C. Arakawa, Turbulence model for the pressure–strain correlation term accounting for compressibility effects. Int. J. Heat Fluid Flow. 21, pp. 354–358 (2000).

[5] S. Huang, S. Fu, Modelling of pressure–strain correlation in compressible turbulent flow. Acta Mech. Sin. 24, pp. 37-43 (2008).

[6] A. Simone, Etude théorique et simulation numérique de la turbulence compressible en présence de cisaillement ou de variation de volume à grande échelle. Thèse de Doctorat, Ecole Centrale de Lyon, France (1995).

[7] M. Riahi, T. Lili, Spectral Modeling and Numerical Simulation of Compressible Homogeneous Sheared Turbulence. Numerical Simulation-From Theory to industry, chapter 18, pp. 393-414 (2012).