International Journal of Mechanics and Energy (IJME)
Vol. 6, Issue 2, 2018, ISSN: 2286-5845

Equilibrium evaluation of the Reynolds anisotropy tensor models in the compressible homogeneous sheared turbulence

Mohamed Riahi*, Marwa Dhaoui and Taieb Lili

Department of Physics, Faculty of Sciences of Tunis (FST), University of Tunis El Manar (UTM), 2092, Tunis, TUNISIA

Abstract: In this work, we are interested in the modeling of homogeneous compressible turbulence sheared using rapid distortion theory (RDT). This theory widely demonstrated its relevance and its utility to develop models of turbulence and increase our comprehension of physical phenomena related to turbulent flows. RDT is used to examine linearity of compressible flows in absence of inertial effects. We will use this approach to evaluate equilibrium states (for high values of non-dimensional times *St*) of the Stefan's models concerning components b_{11}, b_{12} and b_{22} of the Reynolds anisotropy tensor. This evaluation is carried out in the compressible and pressure-released regimes where RDT is validated. A concordance between results given by RDT and models is obtained, except for b_{12} term and this in the compressible regime.

Keywords: Numerical simulation, turbulence, homogeneous, compressible, rapid distortion theory (RDT).

1. Introduction

Several approaches are used in literature to predict turbulent flows. Rapid distortion theory (RDT) is one of these approaches. It remains a linear approach in the modeling of a compressible homogenous sheared turbulence. In a previous work [1], we have used this theory to clarify the physics of the compressible turbulent flows. An analysis of the behavior of different terms appearing in the turbulent kinetic energy and the Reynolds stress equations allowed us to well identify compressibility effects on structure of homogeneous sheared turbulence. Riahi et al. [2] showed that RDT can be used also to study equilibrium states of such a type of turbulence. The objective of this study is to determine numerical solutions of unsteady linearized equations governing double correlations spectra evolution. RDT code developed by authors [2] solves these equations for compressible homogeneous shear turbulent flows. The code is validated by comparing our RDT results with direct numerical simulation (DNS) of Simone et al. [3] and Sarkar [4] for different values of initial gradient Mach number M_{g0}. Sarkar [4] defined this number by $M_g = \frac{Sl}{a}$, where S, l and a denoted respectively the shear rate, an integral lengthscale and the mean sound speed, to quantify compressibility effects for homogeneous shear flow.

In the compressible and pressure released regimes, we will use this validated RDT code to evaluate asymptotic states of Stefan's models [5] for components b_{11}, b_{12} and b_{22} of the Reynolds anisotropy tensor.

In section 2, RDT equations expressed in the spectral space and numerical method used to solve these equations are presented. The Stefan's models [5] for components b_{11}, b_{12} and b_{22} of the Reynolds anisotropy tensor are given in the third part. Section 4 is devoted to compare these compressible models with RDT results. Finally, we finish this work by a conclusion.

*** Corresponding author:** Mohamed Riahi
E-mail: Mohamed.Riahi@fst.rnu.tn

2. Spectral modeling and numerical method

2.1 Equations in the spectral space

We studied a flow which is compressible, homogeneous, shear and turbulent. The linearized equations of continuity and momentum controlling the fluctuating of velocity u_i and pressure p can be written (Simone [6], Simone et al. [3]) as

$$\dot{u}_i + u_j \frac{\partial \overline{U}_i}{\partial x_j} = -\frac{1}{\overline{\rho}}\frac{\partial p}{\partial x_i} + \frac{\lambda+\mu}{\overline{\rho}}\frac{\partial^2 u_j}{\partial x_i \partial x_j} + \frac{\mu}{\overline{\rho}}\frac{\partial^2 u_i}{\partial x_j^2}, \quad (1)$$

$$\left(\frac{\dot{p}}{\gamma \overline{p}}\right) = -u_{i,i}. \quad (2)$$

In the spectral space, the Fourier transform of equations (1) and (2) gave the following equations:

$$\dot{\hat{u}}_i + \frac{\mu}{\overline{\rho}} k^2 \hat{u}_i + \lambda_{ij}\hat{u}_j + \frac{\lambda+\mu}{\overline{\rho}} k_i k_j \hat{u}_j = -I k_i \frac{\hat{p}}{\overline{\rho}}, \quad (3)$$

$$\left(\frac{\dot{\hat{p}}}{\gamma \overline{P}}\right) = -\hat{u}_{i,i}. \quad (4)$$

Where $\lambda_{ij} = S\delta_{i1}\delta_{j2}$ is the mean velocity gradient and $I^2 = -1$.

In the Craya-Herring local reference, the Fourier transform of the velocity field can be expressed as

$$\hat{u}_i(\vec{k},t) = \hat{\varphi}^1(\vec{k},t)e_i^1(\vec{k}) + \hat{\varphi}^2(\vec{k},t)e_i^2(\vec{k}) + \hat{\varphi}^3(\vec{k},t)e_i^3(\vec{k}), \quad (5)$$

where $\hat{\varphi}^1\left(\vec{k},t\right)$ and $\hat{\varphi}^2\left(\vec{k},t\right)$ are the solenoidal modes and $\hat{\varphi}^3\left(\vec{k},t\right)$ is the dilatational mode.

2.2 Application: Case of the pure plane-shear

In the local reference of Craya-Herring, we obtained the following equations system:

$$\dot{\hat{\varphi}}^1 + \upsilon k^2 \hat{\varphi}^1 + \frac{Sk_3}{k}\hat{\varphi}^2 + \frac{Sk_2k_3}{kk'}\hat{\varphi}^3 = 0, \quad (6)$$

$$\dot{\hat{\varphi}}^2 + \left(\upsilon k^2 + \frac{Sk_1k_2}{k^2}\right)\hat{\varphi}^2 + \frac{Sk_1}{k'}\hat{\varphi}^3 = 0, \quad (7)$$

$$\dot{\hat{\varphi}}^3 - 2\frac{Sk_1k'}{k_2}\hat{\varphi}^2 + \left(\frac{4}{3}\upsilon k^2 + \frac{Sk_1k_2}{k^2}\right)\hat{\varphi}^3 + ak\hat{\varphi}^4 = 0, \quad (8)$$

$$\dot{\hat{\varphi}}^4 - ak\hat{\varphi}^3 = 0. \quad (9)$$

Where k_1, k_2 and k_3 are the components of the wave vector $\vec{k}$ and $k' = \sqrt{k_1^2 + k_3^2}$. In the case of the pure plane-shear, S denotes the shear rate ($S = \frac{d\overline{U}_1}{dx_2}$ = constant).

2.3 Doubles correlations

Using the expression of the spectral tensor of the doubles correlations

$$\Phi_{ij}(\vec{k},t) = \frac{1}{2}\left[\hat{\varphi}^{i*}(\vec{k},t)\hat{\varphi}^j(\vec{k},t) + \hat{\varphi}^i(\vec{k},t)\hat{\varphi}^{j*}(\vec{k},t)\right] \quad (10)$$

(where the asterisk represents a complex conjugate), we were able to write evolution equations of these doubles correlations (Riahi et al. [2,7]):

$$\frac{d}{dt}\Phi_{11} = -2\upsilon k^2\Phi_{11} - 2\frac{Sk_3}{k}\Phi_{12} + 2\frac{Sk_2k_3}{kk'}\Phi_{13}, \quad (11)$$

$$\frac{d}{dt}\Phi_{12} = \left(\frac{Sk_1k_2}{k^2} - 2\upsilon k^2\right)\Phi_{12} - \frac{Sk_1}{k'}\Phi_{13} - \frac{Sk_3}{k}\Phi_{22} + \frac{Sk_2k_3}{kk'}\Phi_{23}, \quad (12)$$

$$\frac{d}{dt}\Phi_{13} = 2S\frac{k_1k'}{k^2}\Phi_{12} - \left(\frac{7}{3}\upsilon k^2 + S\frac{k_1k_2}{k^2}\right)\Phi_{13} - ak\Phi_{14} - S\frac{k_3}{k}\Phi_{23} + S\frac{k_2k_3}{kk'}\Phi_{33}, \quad (13)$$

$$\frac{d}{dt}\Phi_{14} = ak\Phi_{13} - \upsilon k^2\Phi_{14} - \frac{Sk_3}{k}\Phi_{24} + \frac{Sk_2k_3}{kk'}\Phi_{34}, \quad (14)$$

$$\frac{d}{dt}\Phi_{22} = \left(-2\upsilon k^2 + 2\frac{Sk_1k_2}{k^2}\right)\Phi_{22} - 2\frac{Sk_1}{k'}\Phi_{23}, \quad (15)$$

$$\frac{d}{dt}\Phi_{23} = 2\frac{Sk_1k'}{k^2}\Phi_{22} - \frac{7}{3}\upsilon k^2\Phi_{23} - ak\Phi_{24} - \frac{Sk_1}{k'}\Phi_{33}, \quad (16)$$

$$\frac{d}{dt}\Phi_{24} = ak\Phi_{23} + \left(\frac{Sk_1k_2}{k^2} - \upsilon k^2\right)\Phi_{24} - \frac{Sk_1}{k'}\Phi_{34}, \quad (17)$$

$$\frac{d}{dt}\Phi_{33} = 4\frac{Sk_1k'}{k^2}\Phi_{23} - 2\left(\frac{4}{3}\upsilon k^2 + \frac{Sk_1k_2}{k^2}\right)\Phi_{33} - 2ak\Phi_{34}, \tag{18}$$

$$\frac{d}{dt}\Phi_{34} = 2\frac{Sk_1k'}{k^2}\Phi_{24} + ak\Phi_{33} - \left(\frac{4}{3}\upsilon k^2 + \frac{Sk_1k_2}{k^2}\right)\Phi_{34} - ak\Phi_{44}, \tag{19}$$

$$\frac{d}{dt}\Phi_{44} = 2ak\Phi_{34}. \tag{20}$$

We used a simple second-order accurate scheme

$$f(t+\Delta t) = f(t) + \Delta t f'(t) + \frac{\Delta t^2}{2} f''(t) \tag{21}$$

(Δt is the time-step size), to determine numerical integration of equations (11)-(20). The derivatives $f'(t)$ and $f''(t)$ are expressed exactly from evolution of theses equations.

3. Presentation of Stefan's models

The formulas of Stefan [5] for components b_{ij} of the Reynolds anisotropy tensor are a function of the gradient Mach number and can be written as follows:

$$b_{11} = \frac{2}{3} - 0.4exp\left(-0.3M_g\right), \tag{22}$$

$$b_{12} = -0.17exp\left(-0.2M_g\right), \tag{23}$$

$$b_{22} = -\frac{1}{3} + 0.17exp\left(-0.3M_g\right), \tag{24}$$

$$\frac{\varepsilon}{SK} = 0.2exp\left(-0.2M_g\right). \tag{25}$$

4. Results and discussion

The code used to evaluate the different models in equilibrium states has been developed and validated by Riahi et al. [2]. Indeed, these authors have shown that states of equilibrium can be studied using rapid distortion theory (RDT) and this in the compressible regime for high values of non-dimensional times *St*; particularly in the pressure-released regime.

Initial conditions that we used are listed in table 1. The two initial gradient Mach number describing the compressible and the pressure-released regimes are respectively $M_{g_0} = 100$ and $M_{g_0} = 1000$. Moreover, numerical simulations are made for two initial turbulent Mach number $M_{t_0} = 0.3$ and $M_{t_0} = 0.4$. In figures 1 and 2, we present evolution of components b_{11}, b_{12} and b_{22} of the Reynolds anisotropy tensor in the compressible ($M_{g_0} = 100$) and the pressure-released ($M_{g_0} = 1000$) regimes. In the compressible regime, equilibrium values of models of Stefan [5] for b_{11} and b_{22} are in good agreement with RDT except the term b_{12} which gives results relatively close to RDT. In the pressure-released regime, a good agreement with RDT is obtained for the different terms b_{11}, b_{12} and b_{22}. Asymptotic values of these terms are independent of initial turbulent Mach number M_{t_0}. In the compressible and the pressure-released regimes, we note that RDT and Stefan's models [5] give the same asymptotic values for components b_{11} and b_{22} which are respectively 0.66 and -0.33. For the term b_{12}, the same equilibrium value is obtained only in the pressure-released regime. We conclude that, only in this last regime, RDT and models of Stefan [5] reproduce correctly asymptotic behaviors of the different terms b_{11}, b_{12} and b_{22}.

Table 1 Initial conditions

Case	M_{g_0}	M_{t_0}
(a)	100	0.3
(b)	1000	0.4

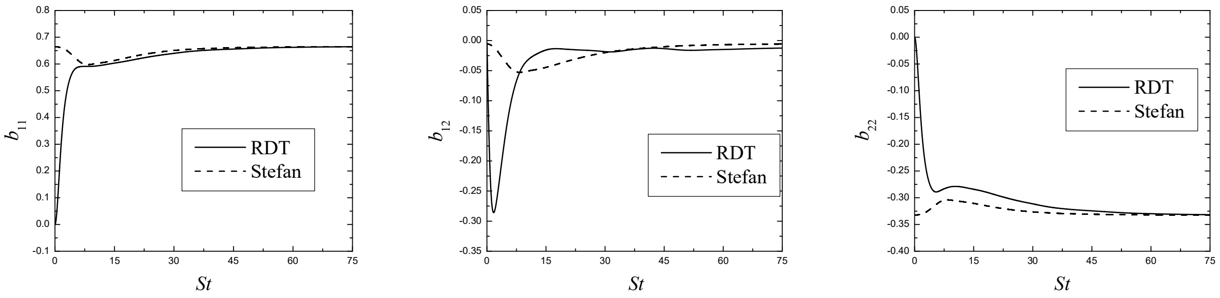

Fig. 1 Evolution of components b_{ij} of the Reynolds anisotropy tensor in case (a).

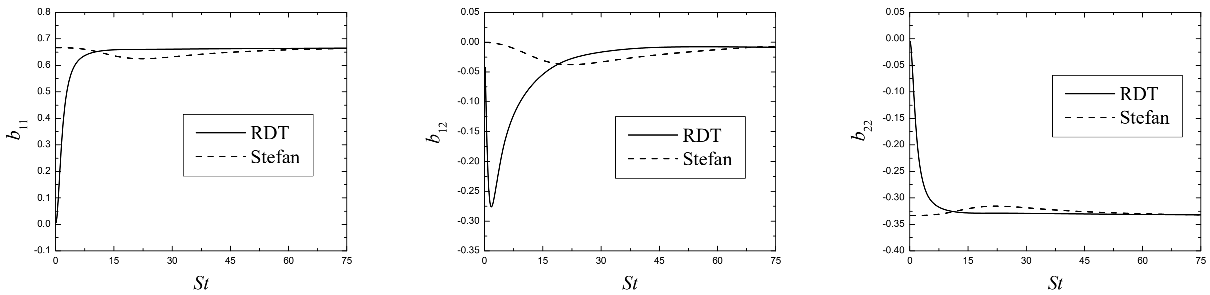

Fig. 2 Evolution of components b_{ij} of the Reynolds anisotropy tensor in case (b).

5. Conclusion

We are interested in the modeling of homogeneous compressible turbulence sheared using rapid distortion theory (RDT). Evolution of compressible homogeneous turbulence has been described completely by finding numerical solutions obtained by solving linear double correlations spectra evolution. Numerical integration of these equations has been carried out using a second-order simple and accurate scheme. In this work, we tested equilibrium states of the compressible models of Stefan [5] concerning components b_{ij} of the Reynolds anisotropy tensor. The different components b_{11}, b_{12} and b_{22} of this model have given results which are compatible with RDT in the compressible $(M_{g_0} = 100)$ and pressure-released $(M_{g_0} = 1000)$ regimes. We note that the deviations remain significant for the term b_{12} in the compressible regime. The equilibrium values of these terms are independent of the initial turbulent Mach number M_{t_0} which is a parameter characterizing the effects of compressibility.

Nomenclature

δ : Dirac delta

M_{g_0} : initial gradient Mach number

M_{t_0} : initial turbulent Mach number

S : shear rate

St : non-dimensional times

l : integral lengthscale

a : mean sound speed

γ : ratio of specific heats

u_i : velocity fluctuation

$\overline{U}$: mean velocity

p : pressure fluctuation

$\overline{P}$: mean pressure

$\overline{\rho}$: mean density

λ : second viscosity coefficient

μ : dynamic viscosity

υ : kinematic viscosity

λ_{ij} : mean velocity gradient

Δt : time-step size

K : turbulent kinetic energy

b_{ij} : anisotropy tensor of Reynolds

ε : total dissipation rate of turbulent kinetic energy

References

[1] M. Riahi, L. Chouchane, T. Lili, A study of compressibility effects on structure of homogeneous sheared turbulence. Eur. Phys. J. Appl. Phys. 39, 67-75 (2007).

[2] M. Riahi, T. Lili, Equilibrium states of homogeneous sheared compressible turbulence, AIP Advances. 1, Issue: 2, pp. 022117-022117-11, ISSN 2158-3226 (2011).

[3] A. Simone, G. N. Coleman, C. Cambon, The effect of compressibility on turbulent shear flow: a rapid-distortion-theory and direct-numerical-simulation study. J. Fluid Mech. 330, pp. 307-338 (1997).

[4] S. Sarkar, The stabilizing effect of compressibility in turbulent shear flow. J. Fluid Mech. 282, 163-186 (1995).

[5] H. Stefan, Statistical Mechanics of Turbulent Flows, Spinger-Verlag, Berlin (2003).

[6] A. Simone, Etude théorique et simulation numérique de la turbulence compressible en présence de cisaillement ou de variation de volume à grande échelle. Thèse de Doctorat, Ecole Centrale de Lyon, France (1995).

[7] M. Riahi, T. Lili, Spectral Modeling and Numerical Simulation of Compressible Homogeneous Sheared Turbulence. Numerical Simulation-From Theory to industry, chapter 18, pp. 393-414 (2012).