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Natural convection for a Herschel-Bulkely fluid inside a differentially heated square cavity

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Abstract: Natural convection for a Herschel-Bulkely fluid, inside a differentially heated square cavity, is studied numerically using Fluent/Ansys 15.0 code. The cavity is heated from the vertical sides and insulated from the horizontal ones. We have studied the effect of Bingham-number (***Bn***) at a fixed Rayleigh-number (***Ra***), then the effect of Rayleigh-number at fixed ***Bn*** and finally the effect of the rheological index ***n*** at fixed ***Bn*** and ***Ra***. Prandtl-number (***Pr***) is taken equal 1.0 for the whole study. Results showed that an increasing ***Bn*** leads to a flow-intensity decrease, and hence its perturbation. Therefore, temperature field becomes less perturbed and the Nusselt decreases. The opposite happens if ***Bn*** decreases. Increasing ***Ra***, leads to similar results as those known for the Newtonian-case, but with a lower intensity because of ***Bn*** effect. The decrease of ***n*** has an opposite effect of that of increasing ***Bn***, and inversely. A rapid tendency toward the conductive problem (***Ra***=0.0) is registered if ***Bn***>0 particularly when ***n***>1.0.

Key words: Natural convection, square cavity, Bingham fluid, Herschel-Bulkely fluid.

1. Introduction

For decades, the world has seen the birth of new products in various fields. The race towards the optimization of the products gave rise to collaborations of several disciplines for the same objective. As an example, electronic parts development requires, in part, the mastery of heat transfer and fluid dynamic processes, since the first enemy of these components is temperature increase as consequence of Joule' effect, which reduces their performance in a first time and risk completely damaging them at a later stage. In another part, the rheological properties of fluids must also be well controlled. These properties directly affect heat transfer processes, by slightly or strongly changing the fluid motion nature (its dynamic), supposed to ensure the heat exchange. Rheology implies that fluid viscosity changes under shear-stress effect, which will sometimes accelerate the flow motion and sometimes decelerate it. Numerous rheological models exist in literature mainly formulated from laboratory tests. A rheological model is a mathematical equation linking shear-stress to fluid deformation, resulting in a non-constant expression of viscosity, which is the feature of non-Newtonian fluids. For this, one can find many rheological laws with different complexity degrees [1]. This complexity made their mathematical and numerical treatments (simulation) in the momentum and energy equations difficult and sometimes still impossible.

The present work deals with natural convection heat transfer in a differentially heated square cavity filled with Herschel-Bulkely fluid. This non-Newtonian fluid is characterized by a flow threshold stress ($\boldsymbol{\tau_0}$) and a rheological index (***n***). Threshold stress characterizes the resistance to deformation or flow. It is only from this value that the flow begins. The rheological index is an exponent in the rheological law of this fluid which expresses the decrease or increase in viscosity of the fluid according to its value (less than or greater than 1.0). This model, proposed in 1926, remains to this day one of the most difficult models to study, especially numerically. The difficulty is caused by the mathematical discontinuity during the passage from a non-deformed area (behaves like a solid) to another in flow. Viscosity is described as infinite (∞) in the first region and dependent on ***n*** in the second.

It should be noted that this type of problem has received a lot of studies in literature it's been more than 60 years. The extensive bibliographic reviews made by Ostrah [2] and Yener et al. [3] enumerated a major part of published works from 1953 to 2013. In addition, such geometry is widely found in practice for Newtonian and Non-Newtonian fluids. Oil-drilling, pulp paper, slurry transport, food processing, polymer engineering, geophysical systems, electronic cooling systems, and nuclear reactors are examples for both types of fluids [4,5].

For works treating the problem close to our study (heated fluid), only few works were found. As example, we can cite the work of Turan et al. [6] and

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Vikhansky [7], in which the authors studied the problem for a Bingham fluid enclosed in a rectangular cavity. Finite difference method were used and they were interested on the effects of ***Ra***, ***Pr*** and ***Bn*** numbers on the dynamic (streamlines) and thermal (isotherms) fields, in addition to the heat transfer coefficient (***Nu***). The geometric ratio (height / width) has also been processed. Boutra et al. [8] have studied the problem in unsteady situation for a square. Lattice Boltzmann method is used for solving equations. The method has shown great efficiency at low shears that usually create numerical instabilities. The vertical walls are maintained at constant and different temperatures while the horizontal walls are assumed adiabatic. The effects of ***Pr***, ***Ra*** and ***Bn*** are treated, where large effects of the three numbers were obtained whether on the dynamic or thermal field. For a steady state situation, Seghier et al. [9] have studied the influence of ***Ra*** on the heat exchange coefficient (***Nu***) by comparing the Newtonian case with the non-Newtonian one. ***Bn*** and ***Pr*** numbers were supposed constant. The authors have shown that ***Nu*** increases with ***Ra*** for both types of fluid, whereas it decreases for the Binghamian case compared to the Newtonian case by reducing the motion intensity under the threshold stress. In addition to the effects of ***Ra*** and ***Bn***, Chakraborty et al. [10] have studied the inclination angle effect applied to a square cavity (in both directions). They found that mean Nusselt number increases with ***Ra*** and decreases with ***Bn***. For large ***Bn*** values, the heating mode becomes almost purely conductive. Inclination angle also has a considerable improving effect in counterclockwise direction. The opposite is recorded in the other direction. Using the finite element method, Huilgol and Kefayati [11] have studied the problem for the same fluid, where they showed that the non-sheared zone (solid zone) is reduced in size as ***Ra*** increases and expands as ***Bn*** increases. This result confirms the findings seen in [10]. In addition, a better heat exchange is recorded with the increase of ***Ra*** and/or the decrease of ***Bn*** by non-sheared zone shrinkage and thus increase of the circulation. It is worth a lot noting that we were unable to find studies close to our present work, dealing with a Herschel-Bulkley fluid as a heating fluid.

2. Problem description

It is a natural convective heat transfer problem inside a square enclosure (Fig.1). The vertical sides are at different temperature (T_h: hot left wall and T_c: cold right wall), while the horizontal ones are insulated. Boussinesq assumption is employed for a laminar regime. Except density, all the remaining parameters are supposed temperature-independent. The heated fluid is of Herschel-Bulkely type ($\tau = \tau_0 + K.\dot{\gamma}^n$), characterized by a yield-shear-stress τ_0 and a flow index ***n***. This rheological model can describe five kinds of fluid following τ_0 and ***n*** values ($\boldsymbol{\tau_0 = 0}$; ***n***=1: *Newtonian fluid*; ***n***<1: *Pseudoplastique fluid*; ***n***>1: *Dilatant fluid*. $\boldsymbol{\tau_0 \neq 0}$; ***n***=1: *Bingham fluid*; ***n***≠1: *Herschel-Bulkely fluid*). We note that this model is sometimes called the generalized Bingham fluid [1]. In this work we have treated only the two last cases when $\boldsymbol{\tau_0 \neq 0}$. For the others, see reference [12].

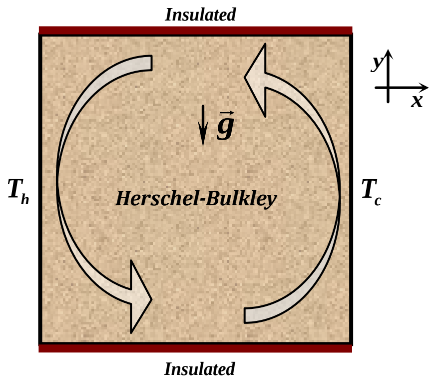

Fig.1 Geometry and problem details

Following the previous considerations, the problem is two dimensional, and its dimensionless governing equations are:

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

x momentum equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\left(\frac{\partial p}{\partial x}\right) + \Pr\left(2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y}\right) + \left(2\frac{\partial \mu_a}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial \mu_a}{\partial y}\frac{\partial u}{\partial y} + \frac{\partial \mu_a}{\partial y}\frac{\partial v}{\partial x}\right) \tag{2}$$

y momentum equation:

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\left(\frac{\partial p}{\partial y}\right) + \Pr\left(2\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y}\right) + \left(\frac{\partial \mu_a}{\partial x}\frac{\partial v}{\partial x} + 2\frac{\partial \mu_a}{\partial y}\frac{\partial v}{\partial y} + \frac{\partial \mu_a}{\partial x}\frac{\partial u}{\partial y}\right) + Ra\Pr\theta \tag{3}$$

where : $\begin{cases} \mu_a = K\dot{\gamma}^{n-1} + \tau_0/\dot{\gamma} & if \quad \tau \geq \tau_0 \\ \dot{\gamma} = 0 \ (\mu_a \to \infty) & if \quad \tau < \tau_0 \end{cases}$

Energy equation:

$$\left(u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y}\right) = \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) \tag{4}$$

Boundary conditions:

$$(x,y)=(0,\,0\rightarrow 1): v=0.0,\ \theta=1$$
$$(x,y)=(1,\,0\rightarrow 1): v=0.0,\ \theta=0 \qquad (5)$$
$$(x,y)=(0\rightarrow 1, 0\ et\ 1): u=0.0\ ;\ \partial\theta/\partial y=0.0$$

The following parameters are used to get the dimensionless equations (stars indicate dimensional):

$$x=\frac{x^*}{L};\ y=\frac{y^*}{L};\ u=\frac{u^*.L}{\alpha};\ v=\frac{v^*.L}{\alpha};\ p=\frac{p^*.L^2}{\rho\alpha^2};\ \theta=\frac{T-T_F}{T_C-T_F}$$
(6)

u , v : Horizontal and vertical velocities respectively;

μ_a : Apparent viscosity;

p,θ : Pressure and temperature respectively;

$Ra=\dfrac{\rho^2 C_P g\beta\Delta T L^3}{\mu_a\lambda}$: Rayleigh number;

$\Pr=\mu_a C_P/\lambda$: Prandtl number;

$Bn=\dfrac{\tau_0}{\mu_a}\sqrt{\dfrac{L}{g\beta\Delta T}}$: Bingham number.

We note that the Bingham number characterizes the yield-shear-stress (resistance to fluid deformation). The resistance to the flow (fluid deformation under stress) is much higher as ***Bn*** is big. When ***Bn***=0, no flow resistance is present.

Since this work treated a heat transfer problem, the Nusselt number is calculated for the studied cases to observe when rheological behavior leads to an improvement or degradation in heat transfer rate. This number is calculated using the following expressions:

$$Nu_i=\left.\frac{\partial\theta_i}{\partial x_i}\right|_{x=0} \qquad (7)$$

$$Nu_{moy}=\frac{1}{N}\sum_{i=1}^{N}Nu_i \qquad (8)$$

Where N is the side nodes number and the index i is the current node where calculation is done.

3. Resolution procedure

Commercial code Fluent/Ansys 15.0 is employed to solve the present problem. Firstly, geometry is plotted under Workbench-Module, exported then to the meshing module, where elements' number, type and fitness are chosen in addition to boundaries nomination and their kinds' definition. Lastly, the geometry and the chosen mesh are then exported to the calculation-Module, where the physical proprieties, the Algorithm of resolution, the accuracy of calculation are fixed.

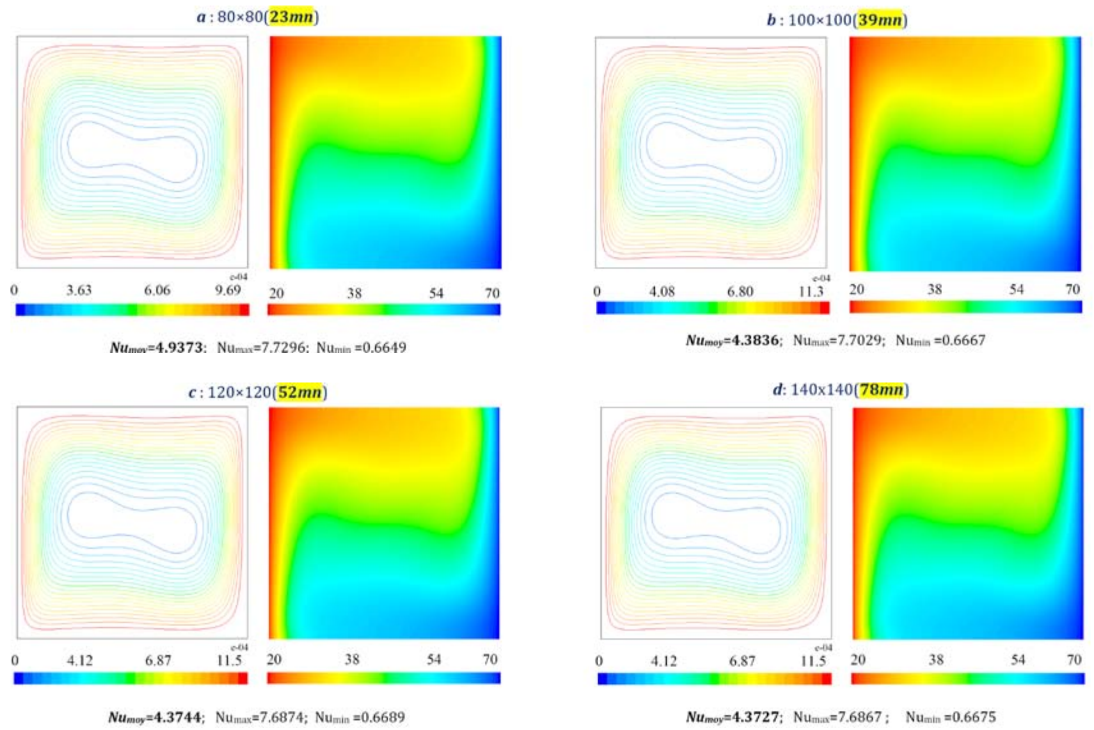

Fig. 2 Mesh effect on stream-function, isotherms and $\mathbf{Nu_{moy}}$; *Ra*=10^{+5}, *Pr*=1 and *Bn*=0.5

The presence of yield-stress in the fluid behavior leads to a singularity between the yielded and the unyielded zones. The fluid behaves like a solid in the second zone and a fluid in the first. The transition from one zone to the other must pass through a separating boundary with infinite viscosity on the solid side, and finite viscosity on the fluid side. Thus a difficulty in the modulating of this phenomenon rises-up at the separating boundary. Different solutions are proposed to handle this constraint [13,14]. The mesh refinement helps also to overcome a little the problem. For this we have made a mesh-results dependency study to choose the optimal one. This part of the work leads to the choice of 120×120 mesh elements with an amplification factor of 1.02 starting from the walls among four tested meshes (Fig.2).

4. Code validation

After choosing the optimal mesh, we have done many validations to be confident with the obtained results. We note that we were not able to find a published work with a Herschel-Bulkely case.

We have presented here three validated cases for:

- A Newtonian fluid (***Bn***=0 ; ***n***=1) with ***Ra***=10^{+5}, ***Pr***=0.71 (Fig.3) with the work in ref. [15];
- A Bingham fluid with different ***Bn***, ***Ra*** and ***Pr*** values (Fig.4) with the work in ref. [11];
- A Bingham fluid with different values of ***Bn*** and ***Ra*** at ***Pr***=1 (Fig.5) with the work in ref. [6]. Good accuracy is found for all the tested cases.

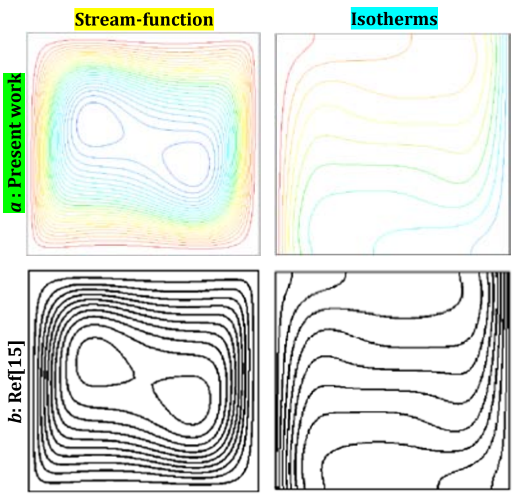

Fig.3 Validation for a Newtonian fluid; *Ra*=10^{+5}, *Pr*=0.71

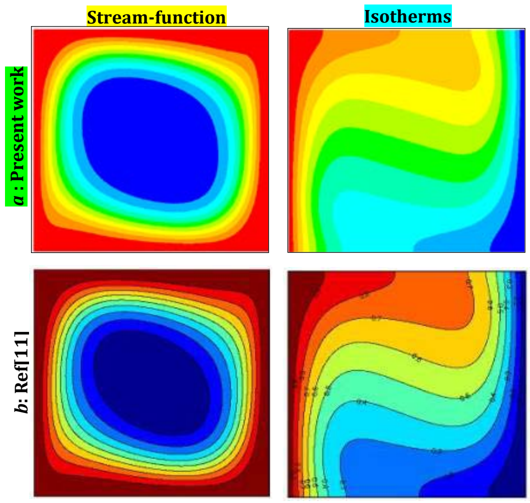

Fig.4 Validation for a Bingham fluid; *Ra*=10^{+5}, *Pr*=1, *Bn*=0.5

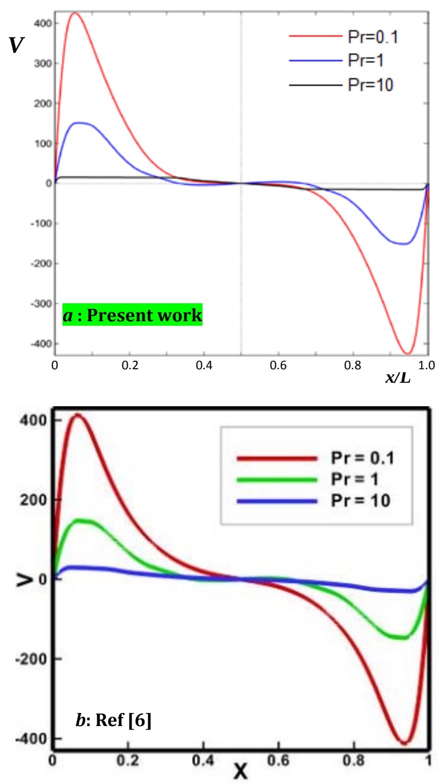

Fig.5 Validation for dimensionless vertical velocity at *y*=0.25*m* for different *Pr*. *Ra*=10^{+5} ; *Bn*=1.

5. Results and discussion

As discussed above, we treat a natural convection problem for a Herschel-Bulkely fluid. This fluid is characterized by a yield-stress represented by the Bingham number (***Bn***) and a rheological index (***n***). Natural convection is characterized by the generation of an ascending flow of hot currents (and descending near the cold wall in our case) with increasing intensity with the value of the Rayleigh number (***Ra***). Therefore, and to better exploit the results, we will classify them in the order:

- Bingham fluid (***n***=1.0) with variable ***Bn*** at ***Ra*** and ***Pr*** fixed: To reveal the effect of ***Bn***;
- Bingham fluid at fixed ***Bn*** and ***Pr*** for variable ***Ra***: To disclose the effect of the flow intensity generated by the buoyancy force;
- Herschel-Bulkely fluids with variable ***n*** (<1 then >1) with fixed ***Ra*** and ***Pr***: To unveil the effect of ***n*** once the others are illustrated.

The effect of ***Pr*** is not treated in this work. In summary it has an opposite effect to that of ***Ra*** [12].

5.1 *Effect of the* **Bn** *number* (***Ra***=10^{+5}; ***Pr***=1.0)

In this part, the values of ***Ra*** and ***Pr*** have been fixed. For the first one the value 10^{+5} is taken. The choice of the ***Ra*** value was based on two things: the first is that it is a high enough value to have an intense natural convection flow; the second is that for this value, two circulating zones are observed for a Newtonian fluid (***Bn***=0.0), which makes it easy to observe the effect of ***Bn*** by making them grow or disappear. It is recalled that for high ***Pr***, one risks losing the two circulating zones since ***Pr*** has an opposite effect to that of ***Ra***.

Before analyzing the results, it should be remembered that the Bingham number reflects the effect of the yield stress. A high ***Bn*** means that the yield-stress to be crossed to have a flow of the fluid is hi gh, the inverse will take place if ***Bn*** is small. For ***Bn***=0.0, the fluid flows under the effect of small buoyancy force (***Ra*** small). Physically, the ***Bn*** will play an opposite role to that of ***Ra***. In other words; if ***Bn*** increases for a fixed value of ***Ra***, after a certain value of ***Bn***, the flow generated by the buoyancy force will stop. This value of ***Bn*** corresponds to the shear-stress generated by ***Ra***. So, for all ***Ra***, we have a ***Bn*** which cancels the motoring shear-stress.

The analysis of Figure (6) showing the current lines for six values of ***Bn*** (0, 1, 2.5, 5, 7.5 and 10) reveals that, flow intensity gradually decreases (see captions). After a certain value of ***Bn***, the two circulating zones disappear and a similar appearance to the case with small ***Ra*** (single circular zone) is recorded. By increasing ***Bn*** again, flow intensity becomes very small. ***Bn***=10 is taken as superior limit since it is largely sufficient to illustrate ***Bn*** effect. Consequently, one can say that after a certain value of ***Bn***, the conduction heating mode (no flow) is reached.

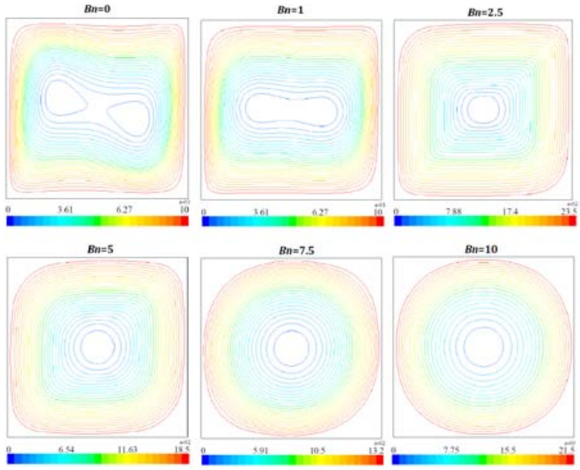

Fig. 6 Effect of *Bn* number on the stream-function; *Pr*=1; *Ra*=10^{+5}

In Figure (7), the effect of ***Bn*** on the temperature field (isotherms) was presented. It is clear that in view of the problem coupling, any disturbance in the velocity field will be recorded in the temperature field. Thus, as the increase in ***Bn*** reduces the intensity of the flow and hence its perturbation, the isothermal field tends to become vertical when ***Bn*** increases. It is recalled that for a pure conduction problem, the isotherms are perfectly vertical.

Figure (8) shows the effect of ***Bn*** on the value of the non-dimensional vertical velocity taken at ***y***=0.25*m*. This position corresponds to the maximum intensity of the velocity ***V***. We can see clearly the reduction of this velocity with the increase of ***Bn***. For example, the value of this velocity is three (3) times greater for ***Bn***=0 than that for ***Bn***=5. It becomes fifteen (15) times greater when ***Bn*** becomes 10. This result implies that ***Bn***'effect intensifies strongly after a certain value.

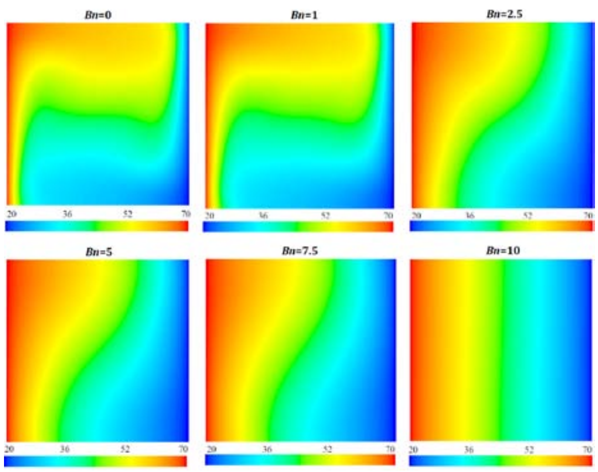

Fig. 7 Effect of *Bn* number on the temperature field; *Pr*=1; *Ra*=10^{+5}

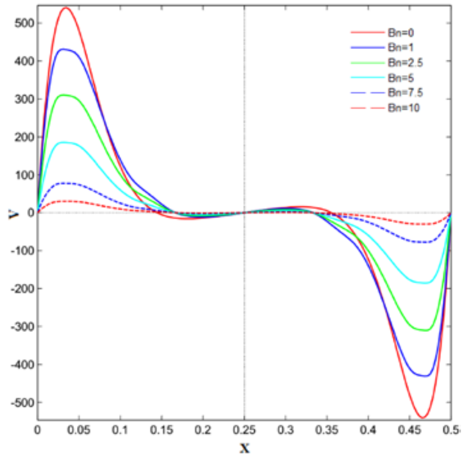

Fig. 8 Vertical velocity profile at *y*=*L*/2 for different *Bn*; *Pr*=1; *Ra*=10^{+5}

From what has been explained, one can conclude that the heat exchange will degrade when ***Bn*** increases. This is the direct cause of the reduction of perturbations in flow and temperature fields (bad mixing and thus bad heat exchange). The values in table (1) confirm this for Nusselt (mean, max and min). By way of indication, the value of Nu_{moy} for ***Bn***=10 is close to that for ***Ra***=10^{+3} for ***Bn***=0.0 [2;5]. The yield-stress (for ***Bn***=10) has reduced the buoyancy force magnitude by a hundred (100) times.

Table 1 Effect of *Bn* on *Nu* number; *Ra*=10^{+5}; *Pr*=1

	Bn					
	0.0	**1.0**	**2.5**	**5.0**	**7.5**	**10.0**
Nu_{moy}	4.5721	3.9965	3.3069	3.1213	2.6255	2.3667
Nu_{Max}	7.9946	7.0038	6.6733	6.1878	5.8933	5.2767
Nu_{Min}	0.7113	0.6333	0.5767	0.57	0.5667	0.5567

*5.2 Effect of the **Ra** number (**Bn**=1; **Pr**=1.0)*

In figure (9) we have presented stream-function and isotherms results for ***Bn***=1 and ***Pr***=1 and four values of ***Ra*** (10^{+3}, 10^{+4}, 10^{+5} and 10^{+6}). It is well known that when ***Ra*** increases, the buoyancy forces increase. As a result, the natural convection flow intensifies and circulating areas are formed for high ***Ra*** values. The reverse occurs when ***Ra*** decreases. This is well known and verified for a Newtonian and power-law fluids [2; 5], when ***Bn*** is nil. When the fluid is a Bingham kind, the yield-stress delays the generation of the flow. The effect of ***Ra*** becomes less intense. It is clear that this phenomenon is clearer as ***Bn*** is large. But, a strong ***Ra*** comes to generate the flow. For this we can see from the figure the opposite effects of ***Bn*** and ***Ra***.

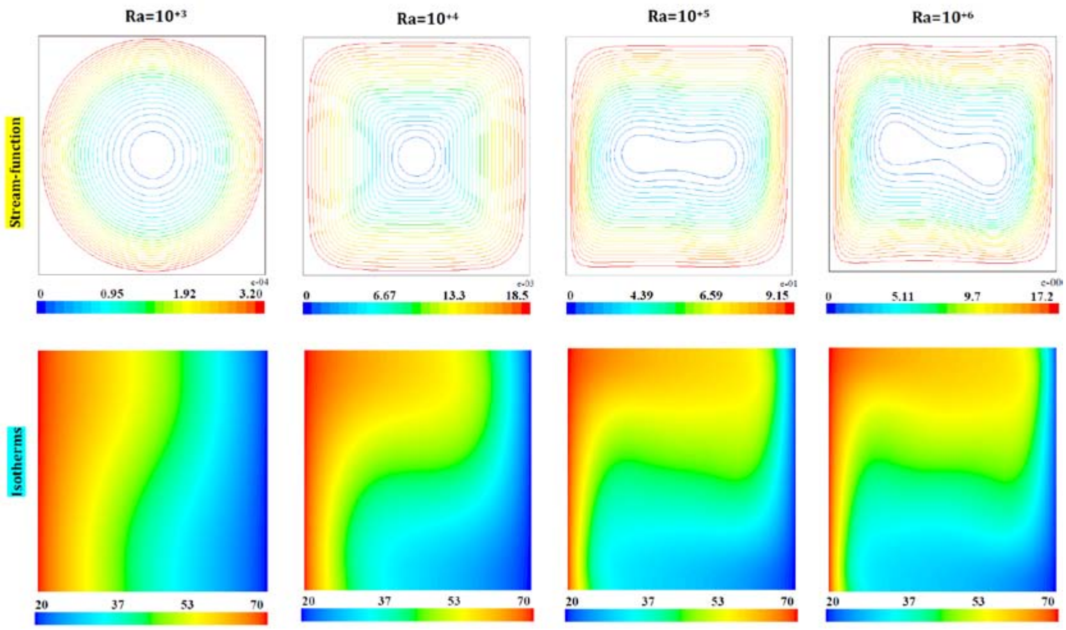

Fig. 9 Effect of *Ra* number on the velocity (stream-function) and temperature fields; *Pr*=1; *Bn*=1

This is well known and verified for a Newtonian and power-law fluids [2; 5], when ***Bn*** is nil. When the fluid is a Bingham kind, the yield-stress delays the generation of the flow. The effect of ***Ra*** becomes less intense. It is clear that this phenomenon is clearer as ***Bn*** is large. But, a strong ***Ra*** comes to generate the flow. For this we can see from the figure the opposite effects of ***Bn*** and ***Ra***.

Compared to ***Bn***=0 (Newtonian Fluid Fig. 6), the effect of ***Ra*** is less important. Therefore, the velocity and temperature fields are less disturbed.

In Figure (10), the effect of ***Ra*** on the vertical velocity ***V*** at ***y***=*L*/2 was presented. As expected, it increases when ***Ra*** increases due to the intensification of buoyancy force. Compared to the Newtonian case (***Bn***=0.0, Fig.8), the registered magnitudes are lower (compare for ***Ra***=10^{+5}).

Since the heat exchange is directly related to the disturbances of the two fields, and this exchange is much better (Nusselt larger) when disturbances are strong, there is an increase in the Nusselt value with ***Ra*** (Table 2). These values are obviously lower than those for ***Bn***=0 and higher if ***Bn*** becomes greater than 1.0 (compare with Table.1).

Table 2 Effect of *Ra* on *Nu* number; *Pr*=1; *Bn*=1

	Ra			
	10^{+3}	**10^{+4}**	**10^{+5}**	**10^{+6}**
Nu_{moy}	1.0985	1.7117	3.9965	7.2597
Nu_{Max}	1.4633	2.6167	7.0038	12.7824
Nu_{Min}	0.5567	0.5833	0.6333	0.74

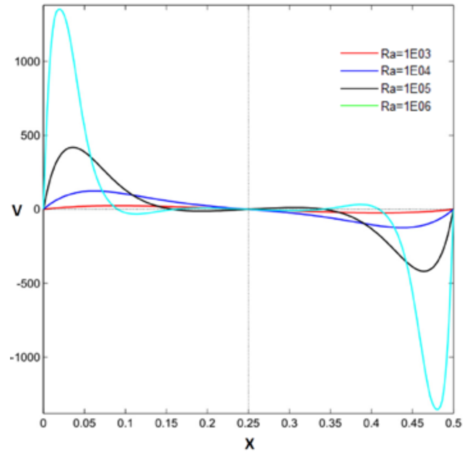

Fig.10 Vertical velocity profile at *y*=*L*/2 for different *Ra*; *Pr*=1; *Bn*=1

5.3 *Effect of the index n* (***Ra***=10^{+5}; ***Bn***=1; ***Pr***=1.0)

In this last part, we will analyze the effect of the rheological index ***n*** for a non-nil yield-stress. Recalling that for ***n***<1, the viscosity of the fluid decreases while for ***n***>1 it increases. This effect, combined with that of ***Bn***, will reduce the braking of the flow when ***n*** is less than 1.0. On the other hand when it is greater than 1.0, braking has been amplified (the braking caused by ***Bn*** will be intensified by the increase of viscosity). So, the Bingham case (***n***=1), can be used to serve as a witness.

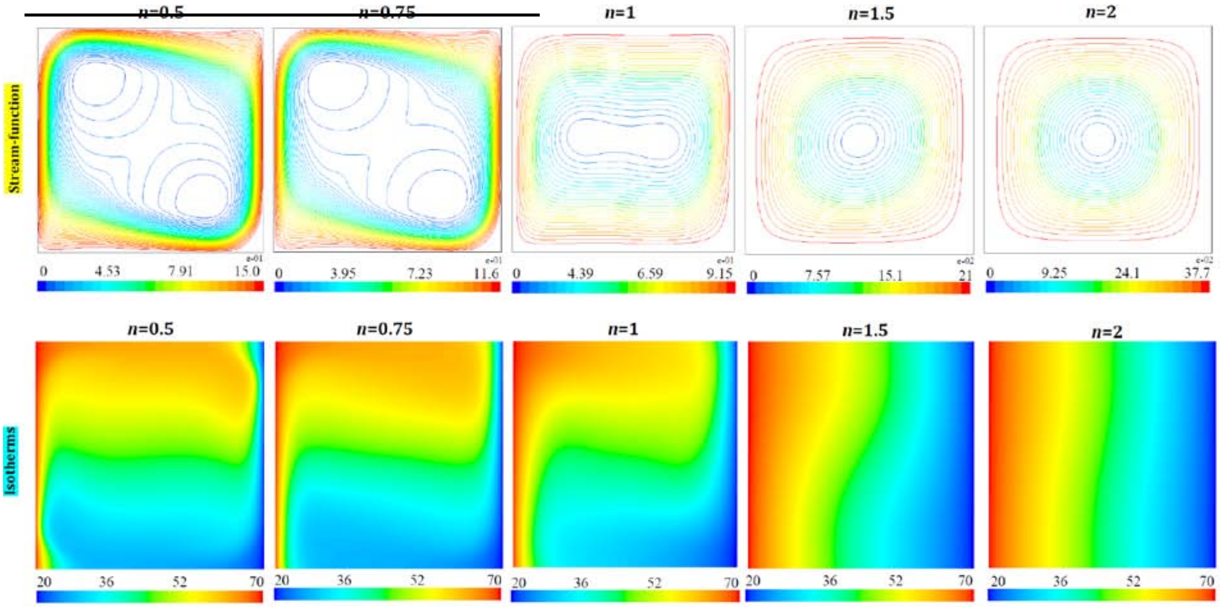

Fig. 11 Effect of index *n* on the velocity (stream-function) and temperature fields; *Ra*=10^{+5}; *Bn*=1; *Pr*=1; *Bn*=1

In figure (11), stream-function and isotherms are presented for five ***n*** values (0.5; 0.75; 1; 1.5 and 2.0). It can easily be seen that the disturbance in both fields (velocity and temperature) increases as ***n*** decreases and decreases when it rises up. This is due to fluid viscosity reduction and hence friction in the first kind (***n***<1). A bizarre circulating zones are formed, indicating the high intensity of the flow. The opposite in the second kind (***n***>1) happened with the disappearance of the circulation zones. Compared to the cases (***Bn***=5; ***Ra***=10^{+5} -Fig.6- and ***Bn***=1; ***Ra***=10^{+4} -Fig.9-), we can see a close results to those when ***n***=2.0. This indicates that the increase of ***n*** has a likewise effect of that of ***Bn*** and an opposite effect of that of ***Ra***. Thus a tendency to the conduction mode is faster as ***n*** increases.

From the above explanation, we can clearly understand the great difference in the vertical velocity magnitudes from ***n***=0.5 to ***n***=2.0 presented in figure (12). For those two results as example, the maximum value registered for ***n***=0.5 is about 18 times that for ***n***=2.0. Furthermore, the first maximum is close to the wall while the second is far from it. This is known as the high velocity increase with fluid shear-thinning (viscosity decrease under shear stress) [1;16].

Consequently, a better heat exchange is found when ***n*** decreases and the opposite when it grows-up as shown in Table 3.

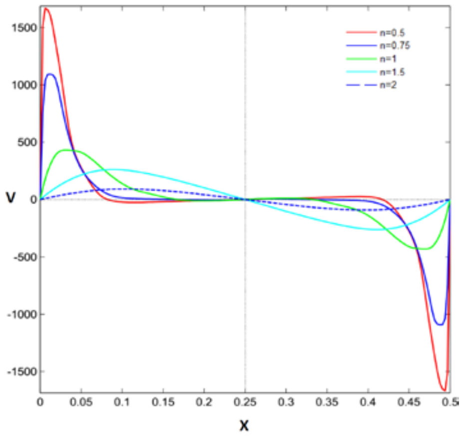

Fig.12 Vertical velocity profile at *y*=*L*/2 for different *n*; *Ra*=10^{+5}; *Pr*=1; *Bn*=1

Table 3 Effect of *n* on *Nu*; *Ra*=10^{+5}; *Pr*=1; *Bn*=1

	n				
	0.5	**0.75**	**1.0**	**1.5**	**2.0**
Nu_{moy}	4.3513	4.1285	3.9965	3.7059	3.5222
Nu_{Max}	7.9946	7.5333	7.0038	6.9867	6.8293
Nu_{Min}	0.7	0.6567	0.6333	0.6133	0.5857

6. Conclusion

The obtained results from this numerical simulation allowed us to conclude that:

- The increase in of Bingham number (***Bn***) increases the resistance to the flow. For a given Rayleigh number (***Ra***), reductions of dynamic and thermal fields perturbations are recorded as ***Bn*** increases;
- Any reduction of the dynamic and thermal disturbances, results in a reduction in the heat transfer intensity (Nu). This may goes until a pure conductive heat mode for high ***Bn*** values;
- For a given ***Bn*** (nil or not), the increase in buoyancy force (***Ra***) improves heat exchange by increasing the dynamic and thermal disturbances;
- The rheological index (***n***) has a remarkable effect and there are two cases:
    - For ***n***<1.0: Decreasing the viscosity when ***n*** decreases reduces the effect of ***Bn***. An improvement in heat exchange results by disturbances amplifications;
    - For ***n***> 1.0: The increase in viscosity (viscous friction braking) when ***n*** increases favors the effect of ***Bn***. Degradation in the Nusselt value is obtained. A faster trend towards the case of pure-conduction (complete blocking) is recorded when ***n*** and ***Bn*** increase at the same time that each independently.

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