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Analysis of stability and control of the longitudinal flight under the influence of longitudinal modes of flight, frequencies and damping coefficients

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Abstract: In this work a comparative study was conducted on the longitudinal dynamics and flight control for four types of aircraft (General aircraft, business aircraft, fighter aircraft and commercial aircraft). We have established the fundamental equations of motion for a rigid aircraft using the laws of mechanical and aerodynamic models. Therefore, an analysis of stability and control is made for every case to determine different dynamic characteristics (longitudinal modes of flight, frequencies and damping coefficients). Also, two kinds of automatic control systems have been designed for four types of aircraft. At the end, some applications are carried on real aircraft models (Cessna 172, M24Learjet, F4C and Boeing 747) and simulation results are presented for each type of aircraft.

Key words: Aircraft, Flight dynamic, Flight control, Stability, Simulation, Modes, Frequency, Damping

1. Introduction

Each type of aircraft is intended to perform a very specific task. Therefore, requirements in terms of stability and maneuverability are different. This is an inverse relationship between these two flying qualities [1]. Stability is most important in the case of a transport aircraft because it does not require rapid maneuvers while the maneuverability is vital to a fighter jet. This work deals with the longitudinal dynamics and control of various kinds of aircraft.

2. Dynamic Model

The establishment of the mathematical model requires the application of Newton's second law to get the equations of motion for a rigid aircraft. Resultant forces acting on aircraft are gravitational, aerodynamic and thrust of propulsion system and similarly, the moments due to these forces about the center of gravity of the aircraft can be expressed in set of equations, as presented by Stevens and Lewis [2] and referenced to the aircraft body frame:

$$\dot{U} = V_R - W_Q - g.sin\theta + \frac{(F_{Ax} + F_{Tx})}{m} \quad (1)$$

$$\dot{V} = -U_R + W_P + g.sin\phi.cos\theta + \frac{(F_{Ay} + F_{Ty})}{m} \quad (2)$$

$$\dot{W} = U_Q - V_P + g.cos\phi.cos\theta + \frac{(F_{Az} + F_{Tz})}{m} \quad (3)$$

$$\dot{P} = \frac{I_{xz}(I_{xx} - I_{yy} + I_{zz})P_Q - (I_{zz}(I_{zz} - I_{yy}) + I_{xz}^2)Q_R + I_{zz}L + I_{xz}N}{(I_{xx}I_{zz} - I_{xz}^2)} \quad (4)$$

$$\dot{Q} = ((I_{zz} - I_{xx})P_R - I_{xz}(P^2 - R^2) + M)/I_{yy} \quad (5)$$

$$\dot{R} = \frac{(I_{xx}(I_{xx} - I_{yy}) + I_{xz}^2)P_Q + I_{xz}(I_{yy} - I_{xx} - I_{zz})Q_R + I_{xz}L + I_{xx}N}{I_{xx}I_{zz} - I_{xz}^2} \quad (6)$$

Where:

u, v, w: Linear velocity components (forward, side and downward velocity respectively)

p, q, r: Angular velocity components (roll, pitch and yaw rate respectively)

F_{AX}, F_{AY}, F_{AZ} : Aerodynamic force components (drag, side force and lift respectively)

F_{TX}, F_{TY}, F_{TZ} : Thrust force components

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LA, MA, NA : Aerodynamic moment components (rolling, pitching and yawing moment)

L_T, M_T, N_T : Thrust moment components (rolling, pitching and yawing moment respectively)

There are three more equations which define the Euler angles:

$$\dot{\phi} = P + \tan\theta\left(Q.\sin\phi + R.\cos\phi\right) \quad (7)$$

$$\dot{\theta} = Q.\cos\phi + R.\sin\phi \quad (8)$$

$$\dot{\psi} = \left(Q.\sin\phi + R.\cos\phi\right)\sec\theta \quad (9)$$

Where:

φ ,θ, ψ: Roll, pitch, and yaw components of the Euler angles, respectively

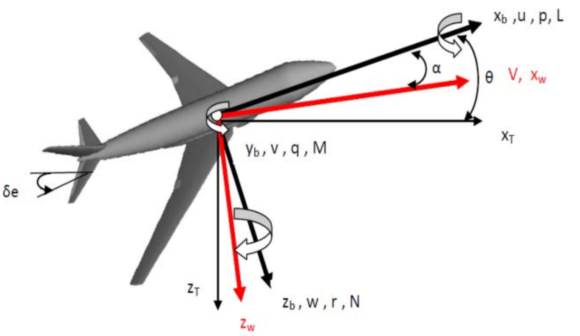

Figure 1. The definition of the axes and variables of an airplane.

After simplification, the linearized equations of longitudinal motion are obtained as [3]:

$$\dot{u} = -g\theta\cos\theta_0 + X_u u + X_{T_u} u + X_\alpha \alpha + X_{\delta e}\delta_e \quad (10)$$

$$U_0\dot{\alpha} - U_0\dot{\theta} = -g\theta\sin\theta_0 + Z_u u + Z_\alpha \alpha + Z_{\dot{\alpha}}\dot{\alpha} + Z_q\dot{\theta} + Z_{\delta e}\delta_e \quad (11)$$

$$\ddot{\theta} = M_u u + M_{T_u} + M_\alpha \alpha + M_{T_\alpha}\alpha + M_{\dot{\alpha}}\dot{\alpha} + M_q\dot{\theta} + M_{\delta e}\delta_e \quad (12)$$

The longitudinal motion states are determined by velocity deviation (u), change in angle of attack (α), pitch rate (q), and change pitch angle (θ). In this case, the input is the elevator deflection (δe). Using the Laplace transform, it is possible to convert a system's time into a frequency domain

$$(s - X_u - X_{T_u})u(s) - X_\alpha \alpha(s) + g.cos\theta_1.\theta(s) = X_{\delta e}\delta_e(s) \quad (13)$$

$$-Z_u u(s) + [s(U_1 - Z_{\dot{\alpha}}) - Z_\alpha]\alpha(s) + [-(Z_q + U_1)s + g.sin\theta_1]\theta(s) = Z_{\delta e}\delta_e(s) \quad (14)$$

$$-(M_u + M_{T_u})u(s) - (M_{\dot{\alpha}}s + M_\alpha + M_{T_\alpha})\alpha(s) + (s^2 - M_q s)\theta(s) = M_{\delta e}\delta_e(s) \quad (15)$$

Writing equations in matrix form [3]:

$$\begin{bmatrix} \left(s - X_u - X_{T_u}\right) & -X_\alpha & g.\cos\theta_1 \\ -Z_u, & \left[s\left(U_1 - Z_{\dot{\alpha}}\right) - Z_\alpha\right], & \left[-\left(Z_q + U_1\right)s + g.\sin\theta_1\right] \\ -\left(M_u + M_{T_u}\right), & -\left(M_{\dot{\alpha}}s + M_\alpha + M_{T_\alpha}\right), & \left(s^2 - M_q s\right) \end{bmatrix} * \left[\frac{u(s)}{\delta_e(s)}, \frac{\alpha(s)}{\delta_e(s)}, \frac{\theta(s)}{\delta_e(s)}\right]^V = \left[X_{\delta_e}, Z_{\delta_e}, M_{\delta_e},\right]^V \quad (16)$$

The speed to elevator transfer function can be expressed by:

$$\frac{u(s)}{\delta_e(s)} = \frac{N_u}{D_1} = \frac{A_u S^3 + B_u s^2 + C_u s + D_u}{A_1 s^4 + B_1 S^3 + C_1 s^2 + D_1 s + E_1} \quad (17)$$

Also, the angle of attack transfer function can be written as:

$$\frac{\alpha(s)}{\delta_e(s)} = \frac{N_\alpha}{D_1} = \frac{A_\alpha S^3 + B_\alpha s^2 + C_\alpha s + D_\alpha}{A_1 s^4 + B_1 S^3 + C_1 s^2 + D_1 s + E_1} \quad (18)$$

The pitch rate transfer function is:

$$\frac{q(s)}{\delta_e(s)} = \frac{N_q}{D_1} = \frac{A_q S^2 + B_q s + C_q}{A_1 s^4 + B_1 S^3 + C_1 s^2 + D_1 s + E_1} \quad (19)$$

As:

$$q(s)=s.\theta(s) \quad (20)$$

Consequently, the pitch angle transfer function is:

$$\frac{\theta(s)}{\delta_e(s)} = \frac{1}{s}\frac{q(s)}{\delta_e(s)} = \frac{1}{s}\frac{A_q S^2 + B_q s + C_q}{\left(A_1 s^4 + B_1 S^3 + C_1 s^2 + D_1 s + E_1\right)} \quad (21)$$

All transfer functions have the same denominator D1. It is designated by the characteristic equation:

$$D_1 = A.s^4 + B.S^3 + C\ .s^2 + D.s + E \quad (22)$$

The coefficients of the polynomial transfer functions are determined from expressions of dimensional coefficients [4] based on the data of table1. The study of the dynamic stability depends entirely of the characteristic equation. When it is equal to zero, it can be put in the following form:

$$(s^2 + 2\xi_{sp}\omega_{n_{sp}} s + \omega_{n_{sp}}^2)(s^2 + 2\xi_{ph}\omega_{n_{ph}} s + \omega_{n_{ph}}^2) = 0 \quad (23)$$

Roots of characteristic equation are:

$$S_{sp} = n_{sp} \pm j\omega_{sp} = \xi_{sp}\omega_{n_{sp}} \pm j\omega_{n_{sp}}\sqrt{1-\xi_{sp}^2} \quad (24)$$

$$S_{ph} = n_{ph} \pm j\omega_{ph} = \xi_{ph}\omega_{n_{ph}} \pm j\omega_{n_{ph}}\sqrt{1-\xi_{sp}^2} \quad (25)$$

Where:

ς : Damping ratio

ωn: Undamped natural frequency

ω: Damped frequency

This means that the transient response of the aircraft consists of two terms, one highly damped and with high frequency called short period mode (sp), and the other is very slowly damped and with low frequency oscillation called phugoid mod (ph) [5].

For a small time, the short period mode dominates the dynamic behavior of the flight of an aircraft, it is the immediate response to the deflection of the elevator and that is the reason we are interested in this mode. The simplified transfer function of the pitch angle is given by:

$$\frac{\theta(s)}{\delta_e(s)} = \frac{\left(M_{\delta e} + \frac{M_{\dot{\alpha}} Z_{\delta e}}{U_1}\right)s + \left(\frac{M_\alpha Z_{\delta e}}{U_1} - \frac{Z_\alpha M_{\delta e}}{U_1}\right)}{s\left[s^2 - \left(M_q + M_{\dot{\alpha}} + \frac{Z_\alpha}{U_1}\right)s + \left(\frac{Z_\alpha M_q}{U_1} - M_\alpha\right)\right]} \quad (26)$$

3. Airplane control

The aircraft pitch motion can be controlled by pitch attitude hold mode system.

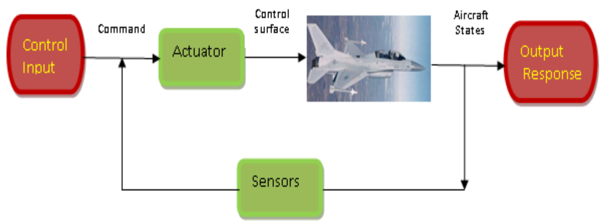

Figure2. The Aircraft control and response

Using Blackelock's approach as shown in the figure 2 and figure 3, the aircraft is initially trimmed to straight and level flight at pitch angle θref , if the output parameter θ varies from the state reference, a signal is generated from the vertical gyro, amplified and directed to the elevator servo. This will actuate the elevator allowing a pitch moment of the aircraft to return to the desired pitch angle. It is a system that does not include an integrator; it is called Type0 fig2 [6].

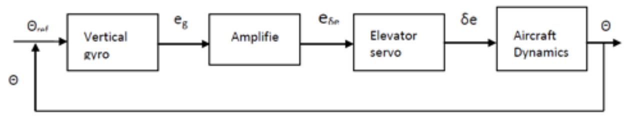

Figure 3.1. The Block-diagram of the Pitch Attitude Hold mode Type0

The improvement of dynamic characteristics of an airplane can be obtained by adding to the previous control system (fig3.1) an inner loop contains a sensor measuring the pitch rate. In this case, the control system is called Type1 (fig3.2).

Table 1. Airplanes Data [4]

Data	Type of Airplane			
	Boeing 747	Learjet M24	Cessna 172	F4C
Flight	conditions			
Altitude H(ft)	20000	40000	5000	35000
Air density: ρ slugs/ft^3)	0.001268	0.000588	0.00205	0.000739
Speed: U1(ft/sec)	673	677	219	876
Initial attitude(rad)	0.0436	0.0471	0.0	0.0454
Geometry	And inertias			
Wing Area: S	5500 ft^2)	230 (ft^2)	174 (ft^2)	530 (ft^2)
Wing chord: c	27.3 ft	7 ft	4.9 ft	16 ft
Wing Span: b	196 ft	34 ft	35.8 ft	38.7 ft
Weight: lbs	564000	13000	2645	39000
Ixx(slugs.ft^2)	13.7e+6	28000	948	250000
Iyy(slugs.ft^2)	30.5e+6	18800	1346	122200
Izz(slugs.ft^2)	43.1e+6	47000	19670	139800
Ixz(slugs.ft^2)	8.3e+6	1300	0	2200
Steady state	coefficients			
C_{L1}	1.76	0.41	0.31	0.26
C_{D1}	0.263	0.0335	0.031	0.03
C_{TX1}	0.263	0.0335	0.031	0.03

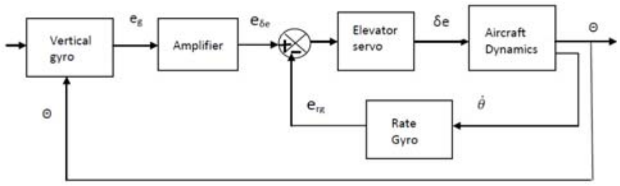

Figure 3.2 The block-diagram of the Pitch Attitude Hold mode with pitch rate feedback Type1

Table 2 Transfer function of the pitch angle for a short period mode

Airplane	Boeing 747	Learjet M24
Transfer function: θ(s)/δe(s)	$\frac{(1.689s+0.8393)}{s(s^2+1.172s+1.587)}$	$\frac{(14.29s+9.137)}{s(s^2+1.99s+8.001)}$
Airplane	Cessna 172	F4C
Transfer function: θ(s)/δe(s)	$\frac{(39.49s+82.04)}{s(s^2+8.331s+37.22)}$	$\frac{(11.4s+5.66)}{s(s^2+1.254s+8.13)}$

Table 3. Dynamics characteristics of short period mode

Airplanes	Undamped natural frequency ωn	Damping ratio ς	Time to half/ double_amplitude
Boeing 747 Learjet M24 Cessna 172 F4C	1.26 2.83 6.1 2.85	0.466 0.352 0.683 0.22	1.18 0.697 0.166 /

Table 4. Dynamics characteristics of phugoid mode

Airplanes	Undamped_natural frequency ωn	Damping ratio ς	Time to half/ Double_amplitude
Boeing 747 Learjet M24	0.0678 0.0914 0.179	0.0297 0.111 0.076	343.86 68.03 50.84

4. Simulation and Result

The Simulation of the dynamic response regarding longitudinal motion of aircraft will be analyzed for four types of airplanes when encountering a small disturbance generated by the deflection of the elevator. The physical data and the aerodynamic coefficients of stabilized flight are presented in table1.

Table 2 shows the transfer functions calculated of short period mode for different types of aircraft used in this study. Also, their dynamic characteristics are calculated for the two modes of motion; short period and Phugoid. They are summarized respectively in Table 3 and Table 4.

4.1 General Aviation Airplane: Cessna 172

Cessna 172 is a small personal transport aircraft with four seats, high wing and single piston engine. For this type of aircraft, simulation results are plotted as curves.. From Figure 4 and figure 5, all the poles are in the left half of the complex plane which means that the aircraft is stable. If the gain exceeds 3.4 then the system becomes unstable. When the design is the type 1, the stability analysis is carried out by plotting the Bode diagram [7], the gain margin as well as increased the phase margin. On the figure 6 and figure 7, the control actuated with a value + 5 (deg) for a time t = 2 to 3 sec.

IV.2. Business airplane: Gates Learjet M24

It is a Gates Learjet business jet aircraft from six to eight seats. This aircraft is equipped with two General Electric jet engines. If the adopted design is made according to the type 1 block, the poles are located on the left side of the complex plane but their location is very close to the origin and the system is stable. When the gain value is increased beyond the value of 3.4, they move to the right, which means that the system becomes unstable. Stability is also analyzed using the Bode diagram, the margins of gain and phase margins (9.85 GM and PM is 41.8 degrees) are satisfactory from the viewpoint of stability. The simulation results for the system type 1 are shown in

Figure 8 and type0 is shown in Figure 9. For the type of system 1, the response time to achieve the desired command (5deg) is about 4 seconds.

4.3. Fighter airplane: McDonnell Douglas F4C

The aircraft F-4C was developed by McDonnell Douglas for U.S. Navy fleet defense which is a supersonic military aircraft with two seats (pilot and navigator) and equipped by twin jet engines of General Electric with afterburner. A fighter aircraft must be high performance especially for maneuverability and response should be fast. The poles are located on the right half of the complex plane, but their position is very close to the origin. The gain value up to 3.5 systems becomes unstable. In type 1 system, the response is oscillatory and the response time is very slow. So this design is not appropriate for fighters because they require a faster response than other aircraft.

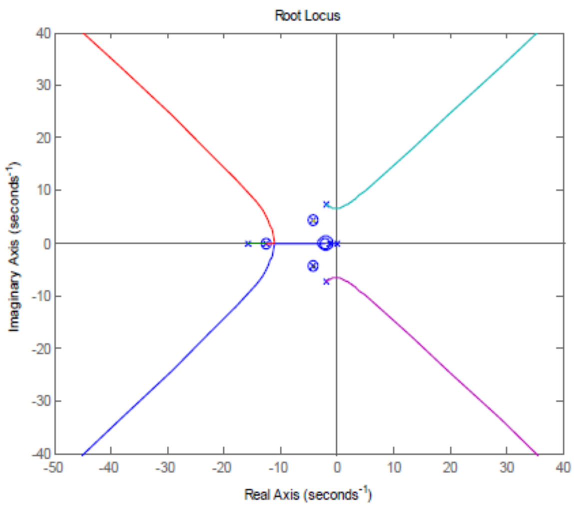

Figure 4. Root Locus for Cessna 172 type 1

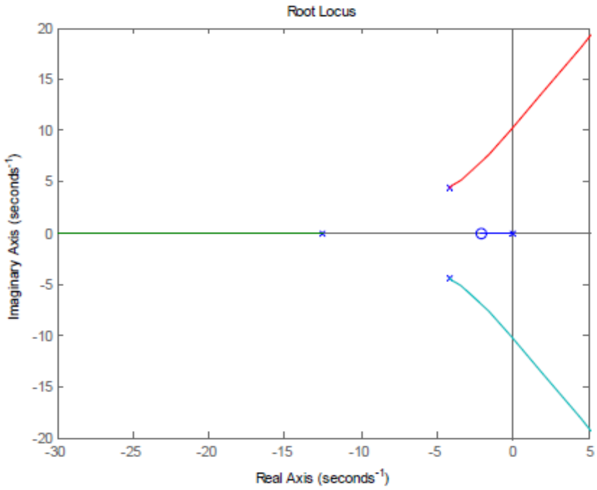

Figure 5. Root Locus for Cessna 172 type 0

4.4. Commercial transport airplane: Boeing 747

Boeing 747 is a four jet engine airliner designed by the American manufacturer Boeing starting from 1965. Depending on the configuration and type of classes, it can accommodate 366-524 passengers. In this case, the poles are located on the left half of the complex plane. This shows that the aircraft is stable. This stability is also analyzed by plotting the Bode plot either for the control system type1. The simulation results of the two types of control system are shown in figure 14.

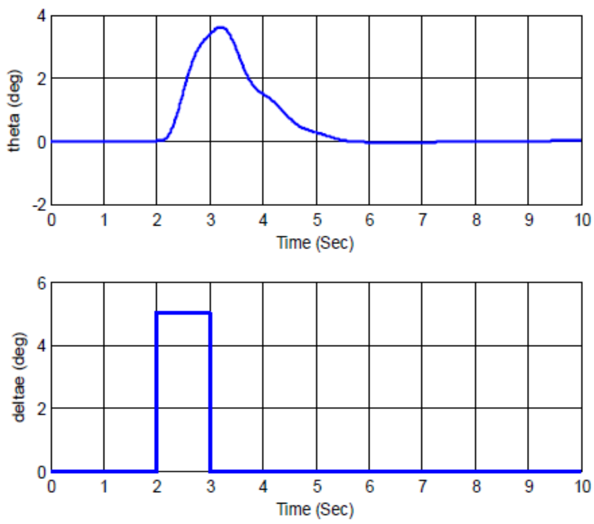

Figure 6. Pitch angle and elevator command for Cessna 172 type 1

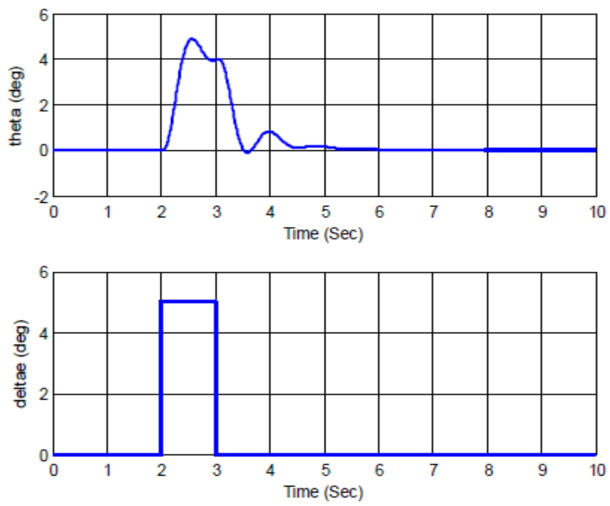

Figure 7. Pitch angle and elevator command for Cessna 172 type 0

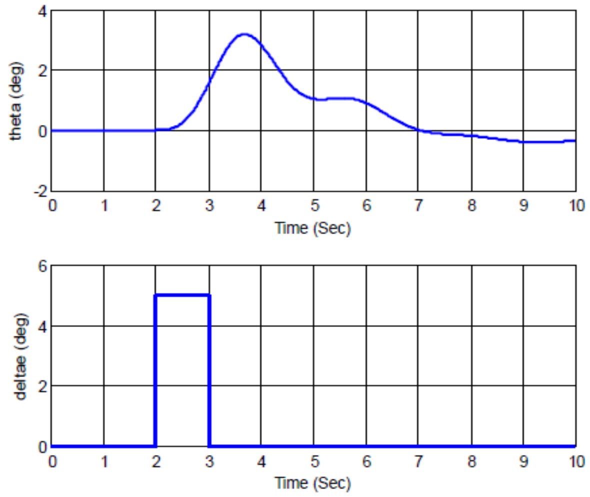

Figure 8. Pitch angle and elevator command for Gates Learjet M24 type 1

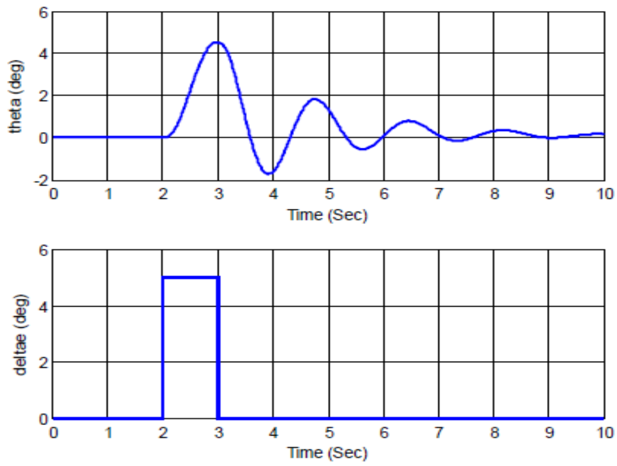

Figure 9.Pitch angle and elevator command for Gates Learjet M24 type 0

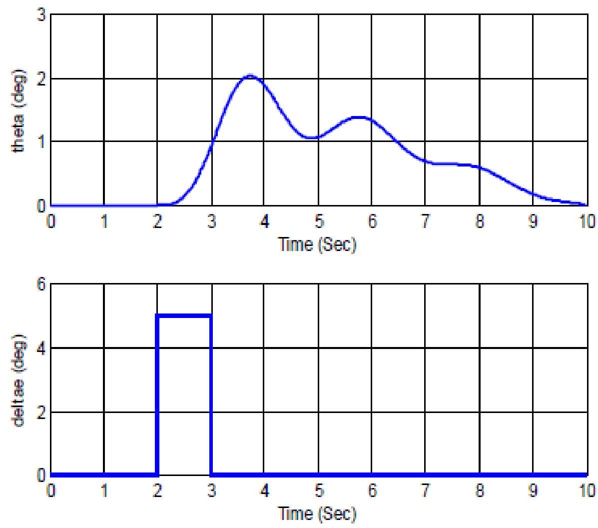

Figure 10.Pitch angle and elevator command for fighter aircraft F4C type1.

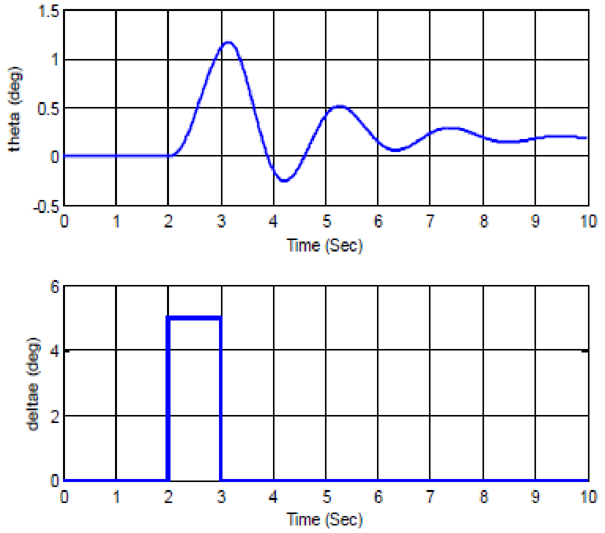

Figure 11.Pitch angle and elevator command for fighter aircraft F4C type0

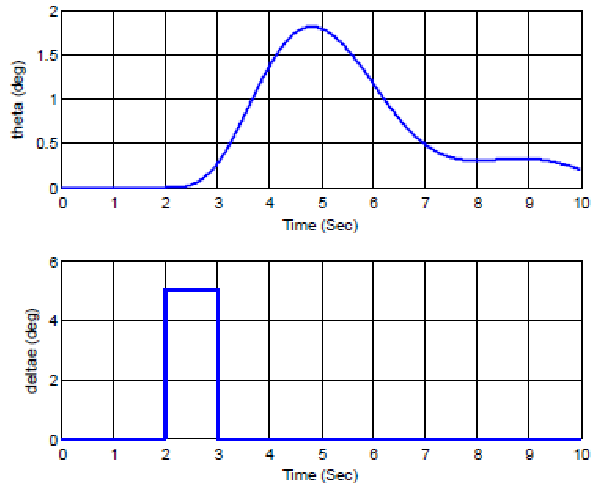

Figure 12.Pitch angle and elevator command for Boeing 747 type1

5. Conclusion

The dynamic characteristics: frequencies, damping factor and the time to reduce by half the amplitude are determined for general aircraft(Cessna172), business aircraft (LearjetM24), fighter aircraft(F4C) and commercial aircraft(Boeing 747), without any control system. To improve the dynamic characteristics of the flight, two kinds of automatic control systems have been designed according to the Blackelock's approach.

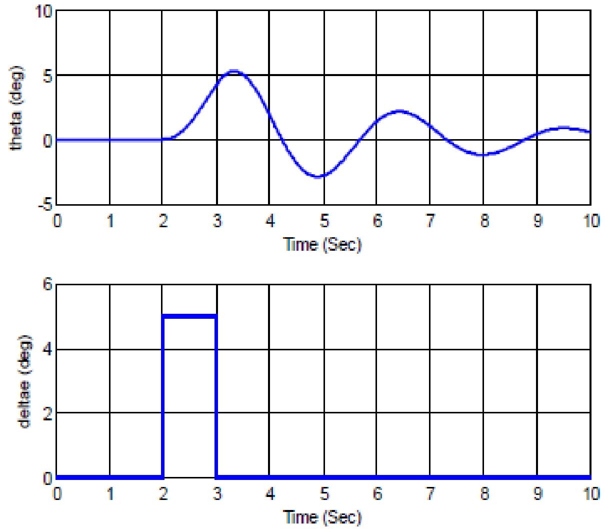

Figure 13.Pitch angle and elevator command for Boeing 747 type 0

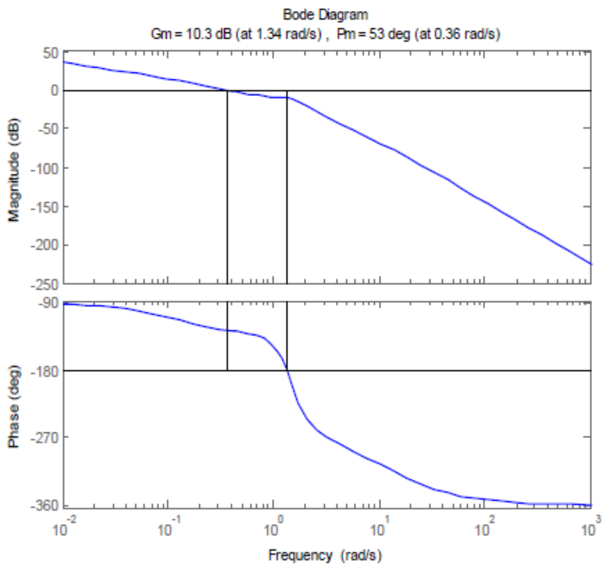

Figure 14. Bode plot for Boeing 747 Type1

The response to the same input control as a pulse has been studied using the same flight automatic control system. For fighter aircraft, the response is oscillatory ,with longer rise time but the response must be faster. The stability of an aircraft depends on its own derivatives stability, its geometrical dimensions, mass and inertial characteristics. Therefore, control system should be done by designing a suitable automatic control which can't be used for all types of aircrafts.

Finally, every aircraft must have its own flight automatic control.

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