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Forced convection of slip-flow in porous micro-duct under Local Thermal Non-Equilibrium conditions

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Abstract: Forced convection heat transfer inside a porous micro duct is performed to investigate the effects of porous material on the heat transfer rate. The flow in the porous material is described by the Darcy- Brinkman- Forchheimer model. The assumption of local thermal non equilibrium (LTNE) condition is adopted for heat transfer and viscous dissipation effects are included in the energy equation of the fluid phase. The obtained governing equations are solved using the Lattice Boltzmann Method (LBM). Knudsen, Eckert and Darcy numbers are selected as the influence parameters. The computational simulations are done for different Knudsen numbers (Kn), Darcy number (Da) and Eckert number (Ec). It is found that both fluid and solid temperatures increases as the Kn decreases, Da decreases and Ec increases. However, the augmentation in Da is related to the important fluid friction effects which decrease the temperature gradient.

Key words: Porous media, Local thermal non-equilibrium, Lattice Boltzmann Method, micro duct.

1. Introduction

The porous medium due to its high thermal conductivity, high specific surface area and good fluid mixing ability has been widely used for heat transfer enchantment in industries [1]. Over the last decades, numerous researchers have studied the fluid flows and heat transfer through porous structures. Many applications linked to porous media have been reviewed and investigated by Nield and Bejan [2]. Heat transfer in a channel filled with porous medium was investigated numerically by Guo and Zhao [3]. Wang et al. [4] analyzed the phenomenon of rarefaction in a micro-annulus filled with a porous medium in the slip-flow regime under local thermal non-equilibrium (LTNE) assumption. Jiang et al. [5] experimentally explored the effect of fluid and porous medium parameters on forced convection in a channel filled with sintered metal. The results indicated that porous media has a profound effect on heat transfer enhancement. Overall, the results show that the use of porous medium can significantly enhance the heat transfer rate.

Various computational methods have been applied to study fluid flows and heat transfer in porous medium. One such numerical method is Lattice Boltzmann Method (LBM). LBM is known as a viable alternative to conventional CFD methods. An important advantage of LBM is handling complex boundaries such as moving (e.g. multiphase) and grossly irregular boundaries (i.e. porous medium) [6]. The LBM has been applied successfully to a lot of fluid dynamics and heat transfer problems, including fluid flows in porous medium, thermal two-phase flow and microparticles transport in a concentric annulus [7–10].

Motivated by the above works, the present paper studies a forced convective heat transfer of fluid flow in a porous micro duct using lattice Boltzmann method. Effects of the Knudsen (Kn), Eckert (Ec) and Darcy (Da) numbers on the heat transfer characteristics are examined.

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2. Mathematical formulation

The study considered a physical model of a two dimensional micro circular cylinder (Fig. 1) filled with a fluid saturated porous medium. The slip-flow regime ($10^{-3} \leq Kn \leq 10^{-1}$) is considered. [11]

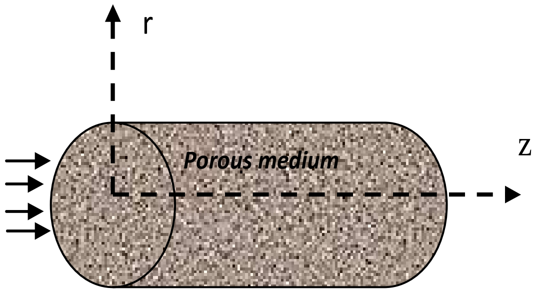

Fig. 1: Geometry and the coordinates system

The following assumptions are considered:

- The flow field is supposed axisymmetric and laminar,
- The porous medium is homogenous, isotropic, saturated with a Newtonian single fluid phase and supposed to be in LTNE,
- The thermo-physical properties of both fluid and solid phases are constant,
- Viscous dissipation effects are included in energy equation for fluid phase,
- The compression effects are neglected,

Continuity equation:

$$\frac{\partial \mathbf{U}}{\partial z}+\frac{1}{r}\frac{\partial (r\mathbf{V})}{\partial r}=0$$

Momentum equation

$$\frac{\partial \mathbf{U}}{\partial \tau}+\frac{1}{\varepsilon}\left[\frac{\partial \mathbf{U}^2}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mathbf{UV}\right)\right]=-\frac{\partial P}{\partial z}+$$
$$\frac{1}{Re}\left\{\frac{\partial^2 \mathbf{U}}{\partial z^2}+\frac{1}{r}\frac{\partial \mathbf{U}}{\partial r}+\frac{\partial^2 \mathbf{U}}{\partial r^2}\right\}-\left(\frac{\varepsilon}{Da\,Re}+\frac{F_\varepsilon \varepsilon}{\sqrt{Da}}\left|\vec{\mathbf{U}}\right|\right)\mathbf{U}$$

$$\frac{\partial \mathbf{V}}{\partial \tau}+\frac{1}{\varepsilon}\left[\frac{\partial \mathbf{VU}}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mathbf{V}^2\right)\right]=-\frac{\partial P}{\partial r}$$
$$+\frac{1}{Re}\left\{\frac{\partial^2 \mathbf{V}}{\partial z^2}+\frac{1}{r}\frac{\partial \mathbf{V}}{\partial r}-\frac{\mathbf{V}}{r^2}+\frac{\partial^2 \mathbf{V}}{\partial r^2}\right\}-\left(\frac{\varepsilon}{Da\,Re}+\frac{F_\varepsilon \varepsilon}{\sqrt{Da}}\left|\vec{\mathbf{U}}\right|\right)\mathbf{V}$$

Energy equation

$$\frac{\partial \theta_f}{\partial \tau}+\left\{\frac{\partial \mathbf{U}\theta_f}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}(rV\theta_f)\right\}=\frac{1}{Pr\,Re}\left[\frac{\partial^2 \theta_f}{\partial z^2}+\frac{1}{r}\frac{\partial \theta_f}{\partial r}+\frac{\partial^2 \theta_f}{\partial r^2}\right]$$
$$+\frac{Bi R_k}{\varepsilon\, Re\, Pr}(\theta_f-\theta_s)+Ec\left(\phi_1+\frac{\phi_2}{Re}\right)$$

where

$$\phi_1=\left(\frac{\varepsilon}{Re\,Da}+\frac{\varepsilon F_\varepsilon}{\sqrt{Da}}\left|\mathbf{U}\right|\right)\left(\left|\mathbf{U}\right|\right)^2$$

$$\phi_2=2\left\{\left(\frac{\partial \mathbf{V}}{\partial r}\right)^2+\left(\frac{\mathbf{V}}{r}\right)^2+\left(\frac{\partial \mathbf{U}}{\partial z}\right)^2\right\}+\left(\frac{\partial \mathbf{V}}{\partial z}+\frac{\partial \mathbf{U}}{\partial r}\right)^2$$

$$\frac{\partial \theta_s}{\partial \tau}=\frac{R_k}{Re\,Pr\,R_c}\left[\frac{\partial^2 \theta_s}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta_s}{\partial r}\right)\right]$$
$$-\frac{1}{(1-\varepsilon)}\frac{Bi R_k}{Re\,Pr\,R_c}(\theta_f-\theta_s)$$

The dimensionless number and parameters appearing in the above equations are: the Darcy number (Da), the Prandtl number (Pr), the Reynolds number (Re), the Eckert number (Ec), the Biot number (Bi), the heat conductivity ratio (R_k) and the heat capacity ratio (R_c) which defined as follows:

$$Da=\frac{K}{R^2},\quad Pr=\frac{\mu_f C_{pf}}{\lambda_f},\quad Re=\frac{Ru_o\rho_f}{\mu_f},\quad Ec=\frac{u_o^2}{C_f\,\Delta T_{ref}},$$

$$Bi=\frac{hR^2}{\lambda_s},\quad R_k=\frac{\lambda_s}{\lambda_f}\quad \text{and}\quad R_c=\frac{(\rho C_p)_s}{(\rho C_p)_f}$$

3. Boundary conditions

Velocity slip

$$U(r=1)=-\frac{2-\sigma_v}{\sigma_v}Kn\left(\frac{\partial U}{\partial r}\right)_w, V(r=1)=0$$

Temperature jump

$$\theta_f(z,r=1)=1-\frac{2-\sigma_T}{\sigma_T}\left[\frac{2\gamma}{(\gamma+1)}\right]\frac{Kn}{Pr}\left(\frac{\partial\theta_f}{\partial r}\right)_w$$

$$\theta_s(z,r=1)=1$$

Symmetry condition

$$\frac{\partial U}{\partial r}(z,r=0)=V(z,r=0)=0,$$
$$\frac{\partial\theta_f}{\partial r}(z,r=0)=\frac{\partial\theta_s}{\partial r}(z,r=0)=0$$

Inlet condition:

$$U=u_o, V=0, \theta_f(z=0,r)=0, \theta_s(z=0,r)=0$$

Outlet condition:

$$\frac{\partial U}{\partial r}=0, \frac{\partial\theta_f}{\partial r}=\frac{\partial\theta_s}{\partial r}=0$$

4. Numerical method

To solve the differential equations system, Lattice Boltzmann Method (LBM) is employed. The LBM is a powerful numerical technique based on the Kinetic theory for simulation of fluid flows and modeling the physics in fluids. Comparing to the conventional CFD methods, the advantages of LBM include simple calculation procedure, simple and efficient implementation, easy and robust handling of complex geometries [7,8].

In this paper, a modified lattice Boltzmann model based on the two-dimensional, nine-velocity lattice-Bhatnagar-Gross-Krook fluid is presented for axisymmetric flows in which the "source" terms was added in the standard 2D Lattice Boltzmann Equation (LBE) [12, 13]. A dynamic and thermal Lattice Boltzmann model with nine velocities, D2Q9, has been used. The developed Lattice Boltzmann equation (LBE) of a density distribution function for the evolution of the velocity field is given as:

$$f_i(\mathbf{z}+\mathbf{e_i}\Delta t, t+\Delta t)-f_i(\mathbf{z},t)=-\frac{1}{\tau_r}\left[f_i(\mathbf{z},t)-f_i^{eq}(\mathbf{z},t)\right]$$
$$+\Delta t S_i(z,t)$$

Where $f_i(z,t)$ is the density distribution function for the velocity field and represents the probability that a particle with velocity e_i is at position x at time t, $f_i^{eq}(x,t)$ is the equilibrium distribution function (EDF), Δt is the time increment and is the dimensionless relaxation time

The equilibrium distribution of D2Q9 model (Huang et al., 2006, El Abrach et al., 2013) is defined as:

$$f_i^{eq}=\omega_i\rho\left[1+\frac{\mathbf{e_i u}}{c_s^2}+\frac{(\mathbf{e_i u})^2}{2c_s^4}-\frac{\mathbf{u}^2}{2c_s^2}\right] \quad i=0,1,2,\dots,8$$

Fluid phase Lattice Boltzmann equation:

$$g_i(z+e_i\Delta t, t+\Delta t)-g_i(z,t)=-\frac{1}{\tau_g}\left[g_i(z,t)-g_i^{eq}(z,t)\right]$$
$$+\Delta t G_i+\Delta t f_i(z,t)q_i+\Delta t Sr_{f,i}$$

Where:

$$g_i^{eq}=\omega_i\theta_f\left[1+\frac{e_i u}{\varepsilon c_s^2}\right]$$

Solid phase Lattice Boltzmann equation:

$$h_i(z+e_i\Delta t, t+\Delta t)-h_i(z,t)=-\frac{1}{\tau_h}\left[h_i(z,t)-h_i^{eq}(z,t)\right]$$
$$+\Delta t Sr_{s,i}$$

Where:

$$h_i^{eq} = \omega_i \theta_s$$

Where $g_i(x,t)$ and $h_i(x,t)$ are respectively fluid and solid temperature distributions functions, τ_g and τ_h are the relaxation times for fluid and solid phases respectively. The $g_i^{eq}(x,t)$ and $h_i^{eq}(x,t)$ are the equilibrium distribution functions corresponding to g_i and h_i .

5. Results and Discussion

The numerical results are given in terms of thermal field distribution as function of Kn, Ec and Da. Examining the graphical results, one can observe that the dimensionless solid and fluid temperatures profiles have the same tendency for different parameters combination.

Also, it can be depicted from fig. 2 that for high values of Kn, solid and fluid temperatures shift down. This can be attributed to two effects: (a) for large values of Kn, mass flow rate through the micro cylinder become relevant (i.e., heat source at the wall must heat greater amount of fluid), (b) as Kn increases the jump in the fluid temperature at the wall increases and this leads to a less amount of heat transfer from the wall to the fluid (Haddad et al., 2005). The same behavior is valid for the solid temperature.

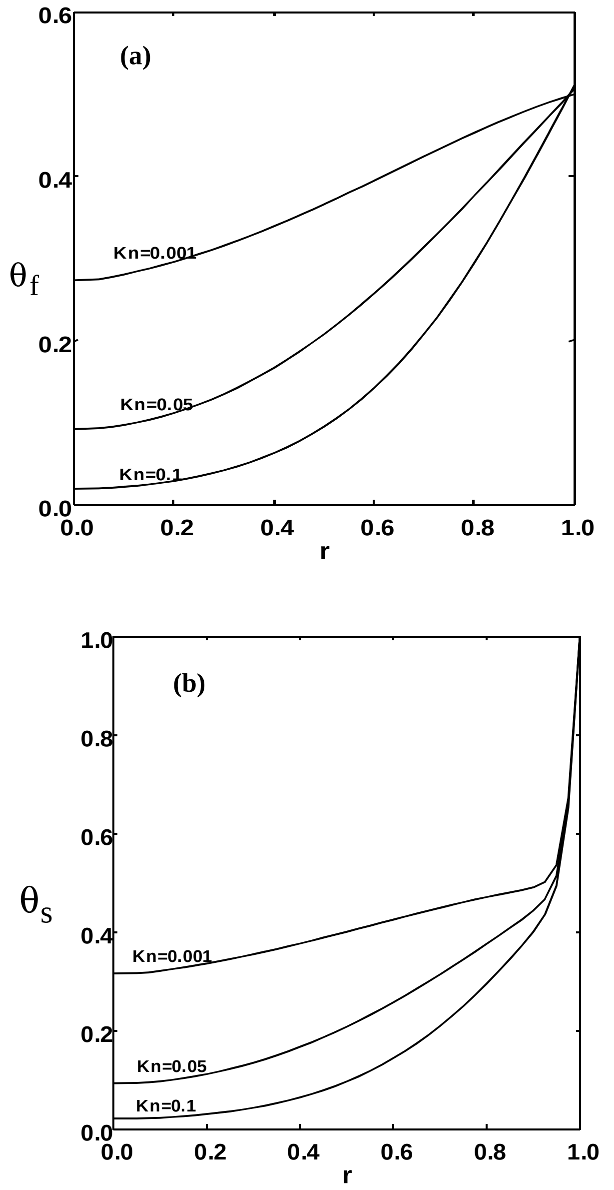

Fig. 2: Effect of Kn on: a) fluid temperature, b) solid temperature.

The contribution of viscous dissipation in the temperature profile is well presented in fig. 3. It can be deduced that high Ec values causes a considerable increase of the temperature.

This outbreak of temperature is due to the additional source of thermal energy (heat) provided by viscous dissipation in the micro duct. The same results are seen for the solid temperature distribution.

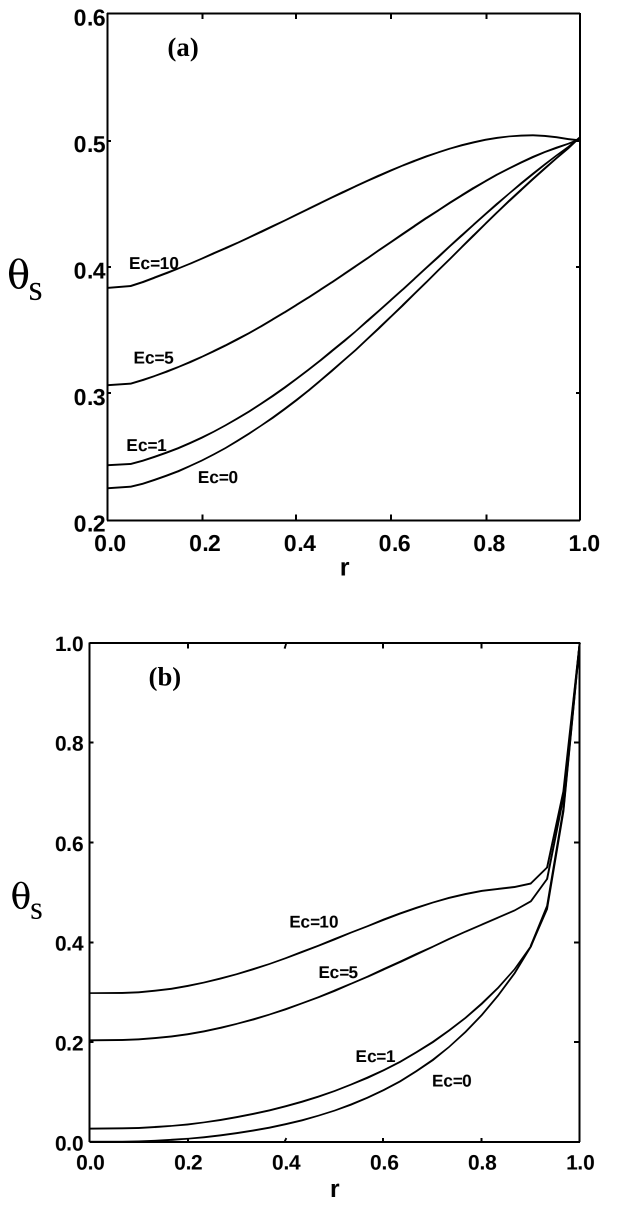

Fig. 3: Effect of Ec on: a) fluid temperature, b) solid temperature.

The effect of the complicated porous medium structure on its thermal analysis is associated with the Da such is intimately related by the medium permeability.

Fig.4 illustrates the effect of Da on solid and fluid temperatures. It is clear that both the solid and fluid temperature distributions increase with the decreases of Da. This is related to the important fluid friction effects which decrease the temperature gradient between the two phases and so enhances the heat transfer to the solid matrix once lessening the permeability.

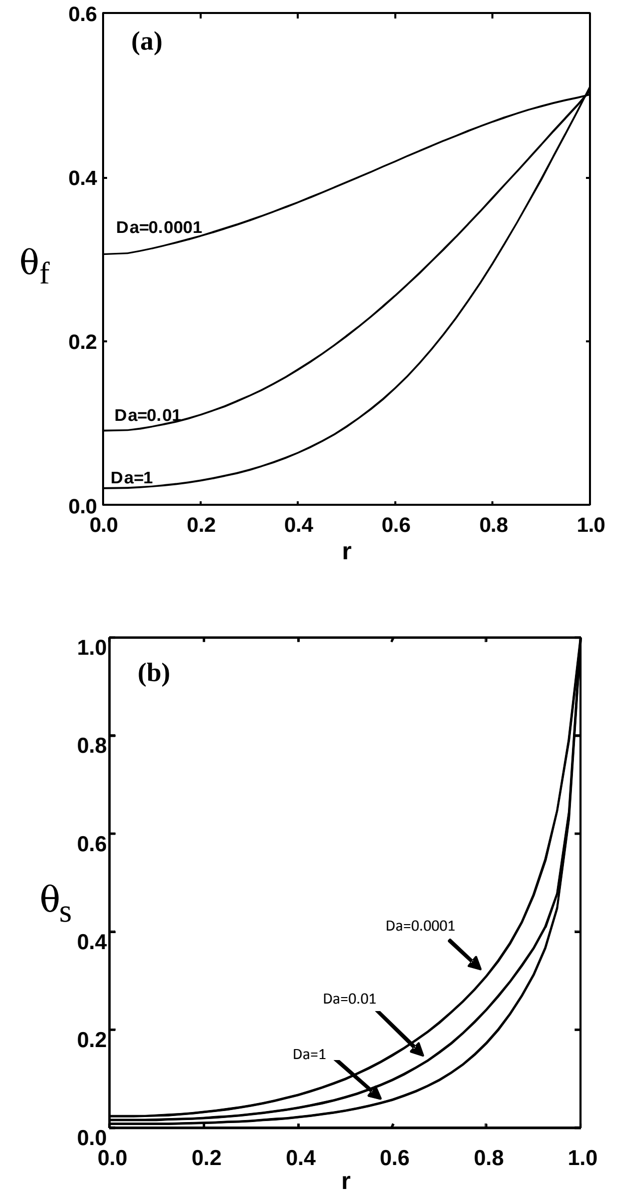

Fig. 4: Effect of Da on: a) fluid temperature, b) solid temperature.

6. Conclusions

This work was undertaken to understand the heat transfer for a thermal non-equilibrium forced convective flow with viscous dissipation effects in a porous micro duct. The results demonstrate that the present modified lattice Boltzmann model is valid for this study. The presence of viscous dissipation effects enhances heat transfer for two phases in the considered system. Furthermore using a porous material with low permeability forwards heat transfer.

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