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Numerical investigation on pressure drop through rectangular microchannels

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Abstract: This paper focuses on investigating, numerically, the laminar single-phase flow through rectangular microchannels with hydraulic diameters varying from 150µm to 550µm, with aspect ratio changing from 0.3 to 0.97. The numerical model employs the three dimensional Navier-Stokes equations to simulate the flow behavior in microchannels. The numerical solution is obtained by discretizing the governing equations using the finite-element method (FEM). The numerical simulations are done on a range of the Reynolds number (Re) varied from 1 to 400. For the isotherm and laminar single-phase flow model, the liquid water is used as the testing fluid. The viscous dissipation, the pressure work and the gravity are neglected. In this study pressure drop shows an agreement with the theoretical results for rectangular channels. However, for Reynolds number range ≤ 100, the friction factor data were found less than the predicted theoretical data presented by Shah and London in rectangular channels.

Key words: Rectangular microchannels, pressure drop, friction factor, single-phase flow.

1. Introduction

Microchannels are one of the essential geometry in microfluidic systems. In literature, different classifications were done to define the microchannels. Obot [1] and Pandey [2] classified the microchannels as the channels having a hydraulic diameter less than 1mm. In the last few decades, these systems have emerged as an important area in research aimed at the development of microdevices. Among various microfluidic systems [microcoolers, microreactors, microbiochips…], rectangular channels are widely used to improve heat transfer, enhance mixed efficiency and shift fluid flow direction [3, 4]. These small devices are currently used in different fields, due to their advantages such as compactness, lightweight and higher surface/volume ratio compared with other macro-scale systems [5]. Therefore, different studies are done in order to investigate the flow behavior through microchannels in regards of the importance of the fundamental understanding of flow characteristics such as velocity distribution and pressure drop in design and process control.

Despite the numerous investigations, contradictory experimental and numerical results exist and lead to questionable discoveries about the micro-effects on the flow characteristics [6, 7].

For the single-phase flow, the studies on pressure drop in microsystems are based on the comparison of the friction factor, f, and the friction constant C*, defined as in equation (1), with their values at macro scales channels.

$$C^* = f.Re \qquad (1)$$

In one part, this comparison showed that the friction constant C* is different from its values in conventional channels. Different experimental studies showed that friction factor, f, in microchannels is less than its values in microchannels [8, 9, 10, 11]. Also, the friction

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constant, C*, so the pressure drop is found higher than the pressure drop in macro scales channels [10, 11, 12, 13, 14]. In another part, other researches showed a good agreement with the theoretical results of pressure drop [15, 16, 17, 18].

Table.1 illustrates the discrepancies of the existing results of the friction factor and pressure drop through microchannels [19].

Table.1 The discrepancies between the existing studies about the friction factor for the single phase flow.

Investigator	$f > f_{theory}$	$f \cong f_{theory}$	$f < f_{theory}$
Wu and Little [12]	✓		
Pfahler et al. [15]		✓	
Choi et al. [8]			✓
Peng et al [9]			✓
Yu et al. [10]	✓		✓
Peng and Peterson [11]	✓		✓
Mala et al [13]	✓		
Lee et Lee [16]		✓	
Faghri and Tumer [17]		✓	
Tu and Hrnjak [14]	✓		
Liu and Garimella[18]		✓	

Numerical studies tried to prove the use of Navier- Stokes equations in describing the flow in microchannels. In addition, numerical studies tried to compare numerical friction factor results with the previous experimental results and theoretical models in macro-scales channels [20, 21].

In this context, a numerical study of the pressure drop in rectangular channels is done in order to investigate the single-phase flow behavior in microchannels. The friction factor, f, values are compared with the conventional theories.

2. Methodology

2.1 Model definition

As presented in Fig.1, the rectangular micro-channel was used for this study as the basic model. wc, hc and L are respectively the width, the depth and the length of the channel.

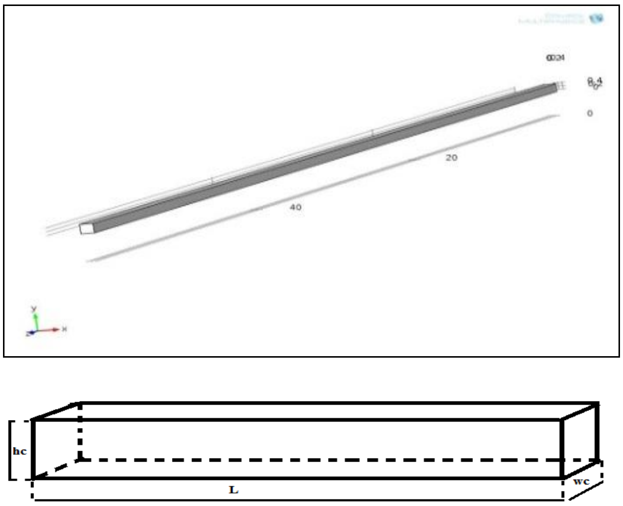

Fig.1 3-Dimension rectangular micro-channel

Each structural parameter is independently adjusted while fixing other variables in order to study its effect on the pressure drop through the microchannel. The investigated parameters are illustrated in Table.2.

Table.2 The basic and investigated structural parameters of microchannel model.

	Variables	Basic	Investigated parameters
Microchannel structural parameters	L (mm)	58	-
	Wc (µm)	555	300-800
	Hc (µm)	488	100-400
	Dh (µm)	519	150-550
	α	0.85	0.13-0.97
	Re	35	1-400

2.2 Mathematical model

In this work, for an incompressible and Newtonian fluid, the mathematical model is based on Navier-Stokes equations. Isothermal conditions are assumed for a laminar single-phase flow and the viscous dissipation, the pressure work, and the gravity are neglected [22]. Therefore, the steady state equations (2, 3) are written as follows:

Mass conservation

$$\nabla\left(\rho.u\right)=0 \tag{2}$$

Momentum conservation

$$\rho.u.\left(\nabla u\right)=-\nabla p+\nabla\mu.\left[\nabla u+\left(\nabla u\right)^{T}\right] \qquad (3)$$

p and u are, respectively, the pressure (Pa) and the velocity (m/s).

This set of non linear equations has been solved according to the following boundary conditions:

- At the microchannel inlet, fluid velocity profile is assumed uniform and constant.
- At the microchannel outlet, no viscous stress condition is chosen and an atmospheric pressure is set.
- At the walls, slip condition is neglected since the working fluid is liquid water with constant properties taken at 293 K: the density is ρ=999.56kg/m3 and μ=0.00101Pa.s is the value of the dynamic viscosity.

The Pressure drop is evaluated through the variation of the friction factor vs. the hydraulic diameter D_h and aspect ratio α and Reynolds number Re, defined respectively in equations (4), (5) and (6) [23].

$$D_h=\frac{2.w_c.h_c}{(w_c+h_c)} \qquad (4)$$

$$\alpha=\frac{h_c}{w_c} \qquad (5)$$

$$Re=\frac{\rho.u_{avg}.D_h}{\mu} \qquad (6)$$

In equation (6), u_{avg} is the average velocity (m/s). The friction factor is calculated as in equation (7):

$$f=\frac{2.D_h.\Delta p}{\rho.L.(u_{avg})^2} \qquad (7)$$

Δp is the pressure difference along the channel considered at the fully developed flow region in order to neglect the entrance effects.

The friction factor can be determined as in equation (8) by the model proposed by Shah and London [23], for laminar incompressible fluid flow in straight rectangular channel.

$$f=(96/Re).\begin{bmatrix}1-3.3553.\alpha+1.9457.\alpha^{2}\\ -1.7012.\alpha^{3}+0.9564.\alpha^{4}-0.2537.\alpha^{5}\end{bmatrix} \qquad (8)$$

2.3 Numerical method and validation

Numerical study of the pressure drop through rectangular microchannel was achieved by the commercial code Comsol Multiphysics using the Finite Elements Method (FEM). In the 3D laminar single-phase flow module, an instructed grid distribution has been used to discretize the computational domain as shown in Fig.2.

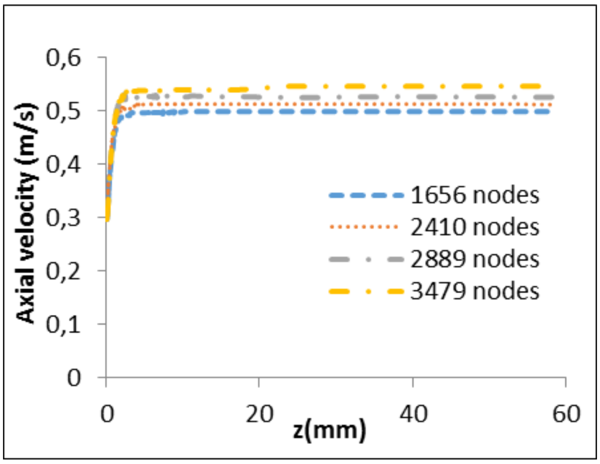

Fig. 2 Grid independent test

Different grid distributions have been tested to ensure that the calculated results are grid independent. The selected grid, for this study, is consisted of 50293 elements (tetrahedral, triangular, prism and quadrilateral) and 1656 nodes. As it is shown in Fig.2, the refinement of the mesh does not change significantly the velocity at the centerline region, and the computational model is stable. In order to reduce the time of resolution, the data are exported to excel and plotted together.

The iterative generalized minimal residual (GMRES) was chosen as a solver for the set of coupled nonlinear differential equations. The solution is assumed to converge when $\left|\frac{\phi^{n+1}-\phi^{n}}{\phi^{n}}\right| \leq 10^{-5}$. This relative tolerance is satisfied for all the independent variables.

3. Results

3.1 Variation of pressure drop with flow rate

Fig.3 presents the pressure drop data of water single-phase flow in rectangular micro-channel with hydraulic diameter of 133 µm. From this figure, it was found that the pressure drop increased with the increasing the flow rate Qv. This behavior is similar to that of the macro-channel laminar flow.

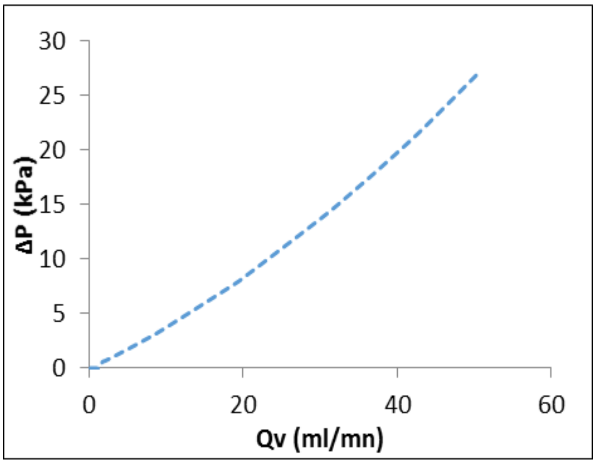

Fig. 3 Variation of pressure drop with the flow rate for rectangular microchannel D_h=133µm.

3.2 Friction factor and friction constant C*=f.Re

The computational data of friction factor are compared to the theoretical data calculated by the correlation of Shah and London formulated in equation (8). The numerical results are shown in Fig.4 and in Fig.5.

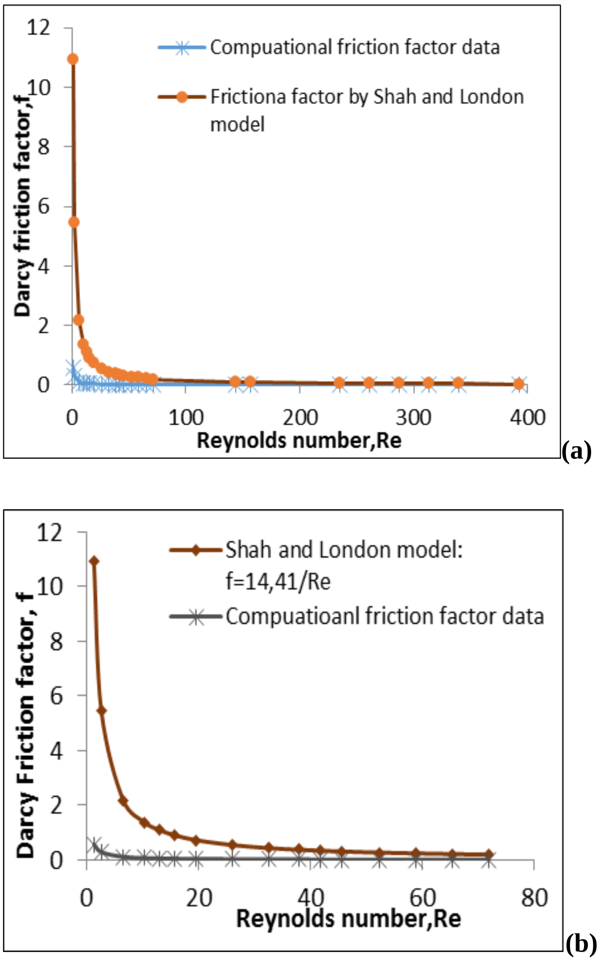

Fig. 4 Friction factor with Reynolds number, Re, in rectangular microchannel with D_h=133µm.

The comparison of the friction factor data, f, for Reynolds number range ≤ 400 is presented in Fig.4.a. It shows that the friction factor has the same behavior as the theoretical friction factor defined in macro-scaled channels, but a difference is important between the two data for the Reynolds number range less than 50 (Re<50), which is detailed in Fig.4.b.

Fig.5 presents the friction factor in the range of the Reynolds number, Re <100 at the form of f versus Re log-log plot presented by Comsol Multiphysics. In laminar single-phase flow, the friction factor is proportional to the Reynolds number. This result is similar to theoretical ones at macroscale channels. The difference between the two data reduces with the increase of the Reynolds number. In rectangular microchannels, for low Reynolds number<100, the friction factor is less than its predicted value in macro-scale channels.

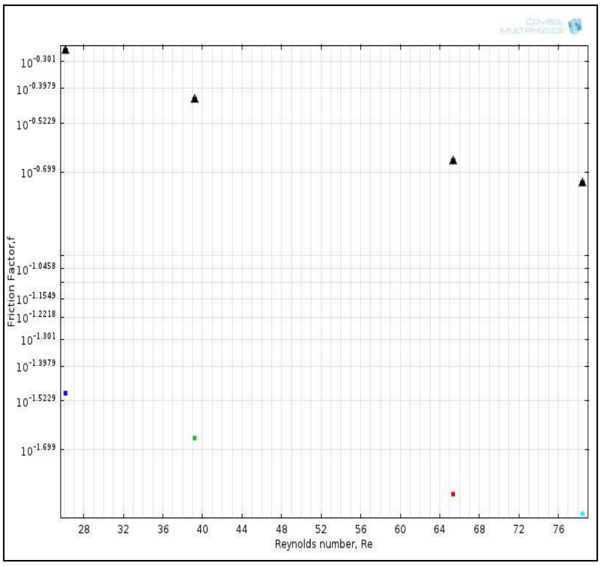

Fig. 5 Friction factor in rectangular microchannel with hydraulic diameter of 519µm for Re< 100: triangle for theoretical data and square for computational data.

3.2 Comparison of friction factor, f, *and friction constant C* versus the aspect ratio,α.*

The numerical friction factor data and friction constant C* as a function of the aspect ratio, α, are presented in the Fig.6 and Fig.7, respectively. For low Reynolds number (Re=35), Fig.6 shows that the friction factor reduces while the aspect ratio increases and is less than the predicted data. Fig.6 presents the friction constant with respect of the aspect ratio for Re=100. Increasing Re, the behavior of the constant f.Re and the data predicted for macro-channels have the same behavior but the computational data is lower than the theoretical data. However, the difference reduces with the increase of Reynolds number, Re, because of the microscale effects.

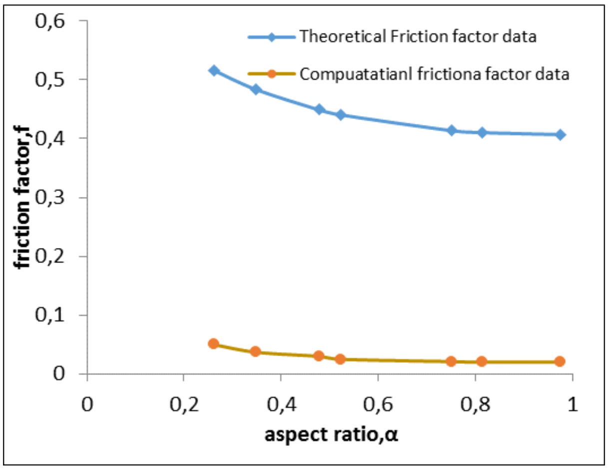

Fig. 6 Friction factor vs. the aspect ratio, α in rectangular microchannel for Re=35

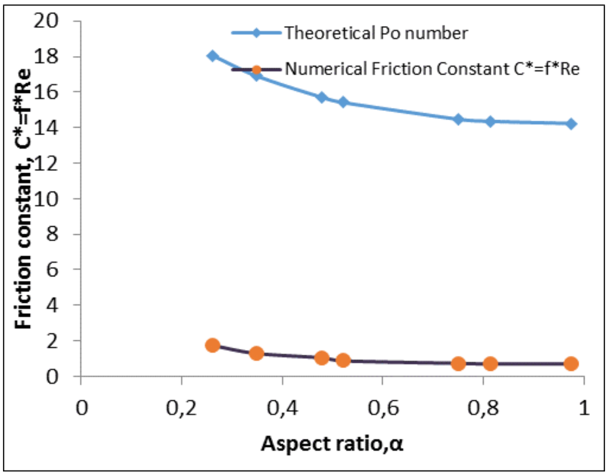

Fig. 7 Friction constant factor, C* vs. aspect ratio, α, in rectangular microchannel for Re=100.

4. Conclusions

For laminar single-phase flow, frictional pressure drops have been computed from the Navier- Stokes equations over a Reynolds number range $1 \leq Re \leq 400$ in rectangular microchannels with the hydraulic diameter range of $150\mu m \leq D_h \leq 550\mu m$, in the aspect ratio range $0.3 \leq \alpha \leq 0.97$. This set of equations is solved by Finite Element Method (FEM) and the solution assumed to converge at a relative tolerance $\leq 10^{-5}$. The following conclusions are obtained after this numerical investigation:

1. Navier-Stokes equations can describe the flow behavior in rectangular microchannels.

2. For lower Reynolds number, the computational data of frictional pressure drop in rectangular micro-channel are less than the predicted data. For higher Reynolds number (Re> 100), the computational data agree well with the theoretical data presented by Shah and London model for macro- scaled rectangular channels.

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