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The effect of Richardson number on thermal and mass behavior of laminar boundary layer flow

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Abstract: The laminar boundary layer is frequently found in nature and in industry. For example: aerodynamic, hydrodynamic, in meteorology and in oceanography. Such as: it causes many problems in aerodynamic such as a jet engine. A numerical study of the boundary layer developing along a horizontal plate is carried out . The aim of this work is to study the effect of Richardson number on thermal and mass behavior of laminar boundary layer, when the flow is subjected to thermal and solutal diffusions.

Key words: Boundary layer, laminar flow, heat transfer, Richardson number, diffusion, friction coefficient.

1. Introduction

The Mixed convection flows are important; they are found in many practical situations as in nature and in man-made devices.

The boundary layer is a thin layer formed when a real flow passes over a solid surface. The velocity of the flow changes through this layer. It increases rapidly from zero at the surface and approaches the velocity of the main stream. Prandtl suggested that the flow may be formed by two parts. The first one is a boundary layer, where the shear stress is the most important. The second part is beyond the boundary layer where the velocity gradient is small and so the effect of viscosity is negligible. In this zone, the flow is essentially of an ideal fluid.

They also differ from isothermal flow due to the induced buoyancy effects via heat transfer. However, the flow and heat transfer properties are more complicated when mixed convection flows are considered past inclined or horizontal plates.

This is due to the fact that the buoyancy forces induce a longitudinal pressure gradient that will directly alter flow and heat transfer rates.

Ref. [1], performed an active control in the case of flow instability on a flat plate. They studied the possibility of controlling the transition zone of the boundary layer by a parietal injection system and periodic aspiration, inspired from theoretical studies of Caller and al. Ref. [1], and experiments conducted by Gad-el-Hak Ref. [1]. In their work, they excited the boundary layer in the transition zone and observed the effect generated in the flow. In the second stage, they realized a parietal control of periodic structures previously created. This test is performed in a wind tunnel, on a flat plate, in the area of the laminar boundary layer in the vicinity of the transition. One remarkable result from this study is that it is possible to act locally in the transition zone of the boundary layer by a system type injection-aspiration parietal. Ref. [2], has done a study of the boundary layer of a plane plate.He noted the sensitivity of convective Tollmien-Schlichting waves and the response to a perturbation localized in space and time.

Ref. [3], has done a numerical study of the thermal and dynamic behavior of a mixed convection laminar

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flow along a vertical plane plate. More precisely, this study examines the influence of Richardson number Ri and the inclination α of the plate on the thermal fields and hydrodynamic flow. She showed in her study that the parameters (Ri, α) has influence on the flow speed, the flow separation, and the thicknesses of the thermal dynamics and heat transfer on the boundary layers.

Ref. [4], have made an experimental study of the interaction between a laminar boundary layer and an open cavity for moderate Reynolds numbers. In order to highlight the three-dimensional structures of the flow, they have used the particle image velocimetry (PIV), the laser Doppler velocimetry (LDV), and the visualization in two planes parallel illuminated with different wave lengths.

Ref. [5], has conducted a numerical study of the permanent and laminar convection heat and mass in the boundary layer along a vertical surface with a force opposite floatability. He aimed to determine more precisely the hydrodynamic fields, heat and mass, and characterize the variation of heat and mass transfer. The simulations were performed with heat and mass Grashof numbers varying between 10^4 and 10^7.Corresponding to a laminar regime in the computational domain. He showed that the increase in buoyant force N leads to an increase of these dimensionless numbers.

The steady laminar flow and thermal characteristics of a continuously moving vertical sheet of extruded material are studied close to and far downstream from the extrusion slot by Ref [6]. The velocity and temperature variations, obtained by finite volume method, are used to map out the entire forced, mixed and natural convection regimes. Regimes of forced, mixed and natural convection have been delineated, in buoyancy assisting flows, as a function of Reynolds and Grashof numbers for different values of Prandtl [6] number and buoyancy force parameter. Ref [6], showed that at low local Richardson number Ri_x, the heat transfer rates due to different convection modes are all equal. This is attributed to the fact that diffusion dominates over both inertia and buoyancy in the region close to the extrusion slot. The heat transfer rate drops sharply with distance in the non-similar forced-convection dominated region. The drop continues downstream but at a slower rate as the thermal boundary layer thickens.

Air flowing past a solid surface will stick the surface. Ref [7] shows that this phenomenon caused by viscosity is a description of the no-slip condition. This condition states that the velocity of the fluid at the solid surface equals the velocity of that surface. The result of this condition is that a boundary layer is formed in which the relative velocity varies from zero at the wall to the value of the relative velocity at some distance from the wall. The goal of the present research of Ref [7] is to measure the velocity profile in the thin boundary layer of a flat plate at zero angle of attack at Reynolds numbers up to 140,000. The measured velocity profiles are compared with results from theory.

2. Mathematic and numerical methods

2.1 Mathematic method

The laminar two-dimensional motion of fluid past a hot semi-infinite plate is considered, with the free stream velocity and temperature denoted by, U_∞ and T_∞. The equations used in this work are the following:

Conservation equation: $$\frac{\partial u}{\partial x} + \frac{\partial V}{\partial y} = 0 \qquad (1)$$

Equation of momentum:

According to (ox):

$$u\frac{\partial u}{\partial x} + V\frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \left(\frac{Gr_T}{\mathrm{Re}^2}T + \frac{Gr_M}{\mathrm{Re}^2}C\right)\cos\alpha \qquad (2)$$

According to (oy):

$$u\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}=-\frac{\partial P}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}\right)+\left(\frac{Gr_T}{\mathrm{Re}^2}T+\frac{Gr_M}{\mathrm{Re}^2}C\right)\sin\alpha \quad (3)$$

Energy equation

$$u\frac{\partial T}{\partial x}+V\frac{\partial T}{\partial y}=\frac{1}{\mathrm{Pr}.\mathrm{Re}}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \quad (4)$$

Diffusion equation:

$$u\frac{\partial C}{\partial x}+V\frac{\partial C}{\partial y}=\frac{1}{Sc.\mathrm{Re}}\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right) \quad (5)$$

Where the x and y variables are, respectively, the horizontal and vertical coordinates, u and v are, respectively, the horizontal and vertical fluid velocities and p is the fluid pressure and $\alpha=(\vec{g},\dot{i})$.

2.2 Simulation model

The direct numerical simulations (DNS) are based on the vorticity-velocity formulation of the complete Navier-Stokes equations for incompressible flat plate flow with streamwise pressure gradient. The equations of conservation of momentum and energy are discretized with the method of finite volumes to shifted mesh. The temporal scheme adopted for solving the above system equations is in type finite difference of second order. All equations of momentum, conservation and energy are discretized by the finite volume method on a shifted grid. Each grandeur is defined on an own mesh.

3. Geometric configuration and boundary conditions

A horizontal plane plate is considered with length L = 10 and width H = 1, who it is subjected to a forced flow parallel to its surface (Fig. 1). The physical characteristic values are:

T_∞ is the ambient temperature; T_p is the temperature of the plate; U_∞ is the velocity at the infinity. The Reynolds number range is from 100 to 1000. The mesh used for most simulations is 200x60. The time step is taken equal to 0.01, and the convergence test is optimised to 10^{-8}. The initial condition imposed is uniform along the entire length of the boundary layer L and width H.

$\frac{\partial^2 u}{\partial x^2}=0$: This boundary condition is presented as a less binding type of Newman condition, particularly with regard to vortex structures flows traverse the output of border. It is more effective in this particular type of flow. This condition can be physically translated as a linear extrapolation of the longitudinal speed output.

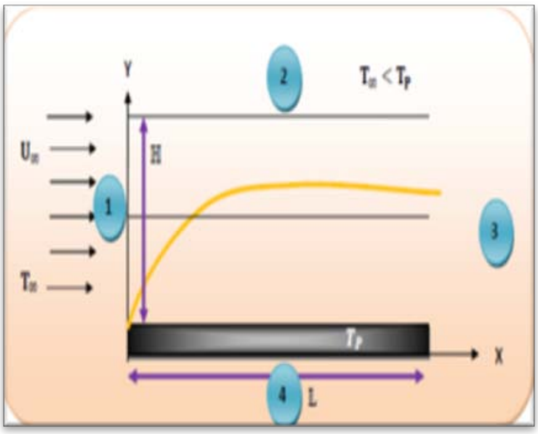


Fig.1 Geometric configuration

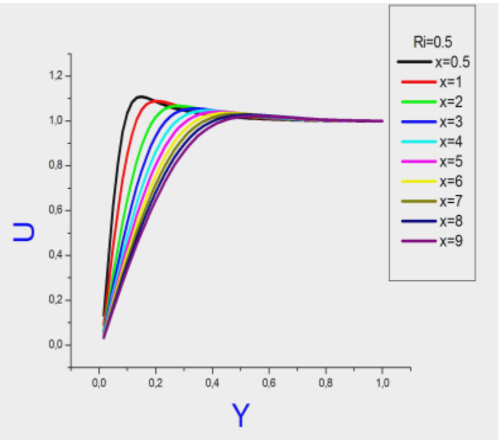
Table 1 Boundary condition

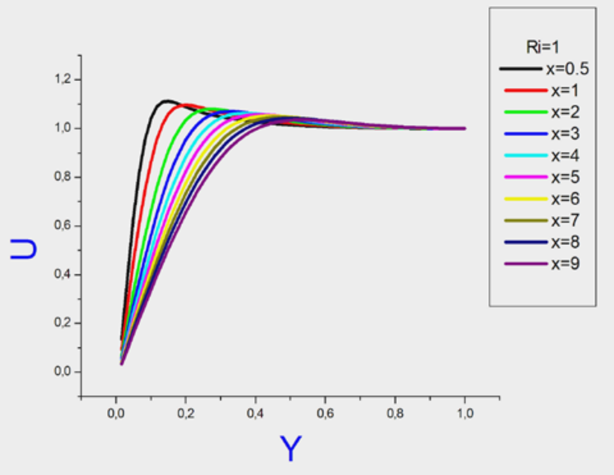
At the entrance (1)	a free upper boundary(2)	the output (3)	a wall(4)
$\frac{\partial T}{\partial x}=0$ $u=u_\infty(\frac{y}{H})^2$ $\vartheta=0$ $c=1$	$u=\vartheta=0$ $T=0$ $\frac{\partial C}{\partial y}=0$	$\frac{\partial^2 u}{\partial x^2}=0$ $\frac{\partial \vartheta}{\partial x}=0$ $\frac{\partial T}{\partial x}=0$ $\frac{\partial C}{\partial x}=0$	$u=0$ $\vartheta=0$ $T=1$ $\frac{\partial C}{\partial y}=0$

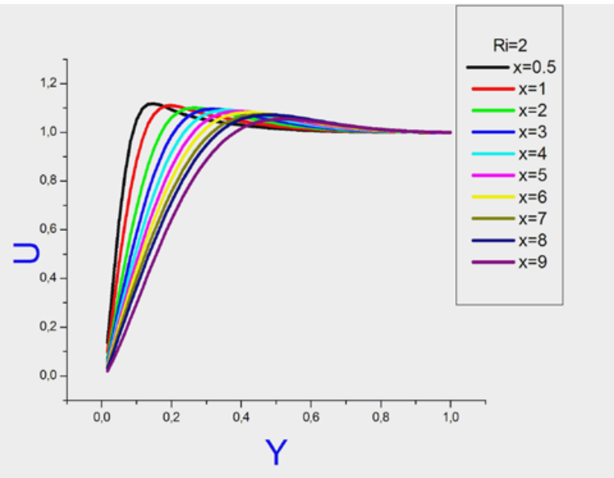
5. Result and discussion

*The effect of Richardson number in laminar flow:

The temperature gradient amplifies the phenomenon of natural convection. It is noted that the increase of Richardson number accelerate the flow near of the wall (fig. 2). This result is explained by the fact that the natural convection increases the speed of the fluid particles in the vicinity of the wall as [3]. This acceleration results in an increase of the thickness of the dynamic boundary layer (Fig. 3) and thus velocity gradients parietal are important where heat transfer parietal increases similar to [7]. The Richardson number value imposed is Ri = 0, then the flow is due to a forced natural convection. This result shows that natural convection helps to accelerate and increase the heat exchange with the wall. For small values of the Richardson number, the forced flow remains dominant across the plate approximating to [6]. It is observed that the mixed convection occurs only from Ri = 0.5 where the speed in the axis becomes more important than the rate imposed on the entry.







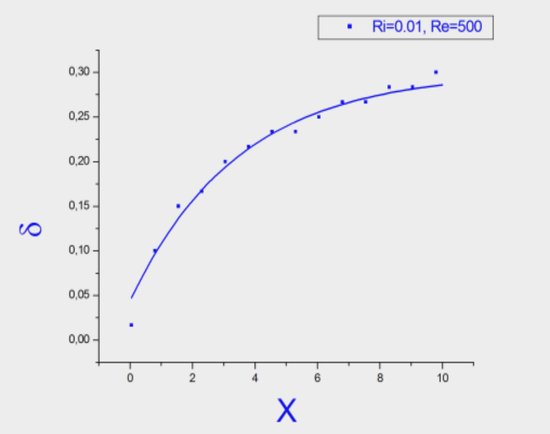


Fig. 3 Evolution of the thickness of the boundary layer dynamic

The increasing of the Richardson number Ri, causes a decrease in the thickness of the thermal boundary layer. This is due to an important parietal thermal gradient.

The friction coefficient decreases if the Richardson number increases (Fig. 4). In fact, the contribution of the natural convection is increasing the velocity and temperature gradient near of the wall like [3]. The initial value of the parameter $\frac{1}{2}CfRe^{\frac{1}{2}}$ is approximately 0.35 similar to that given by Blasius [3] (0.332) value similar to [7].

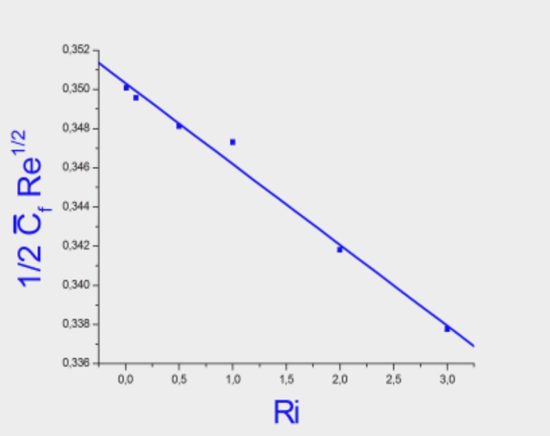
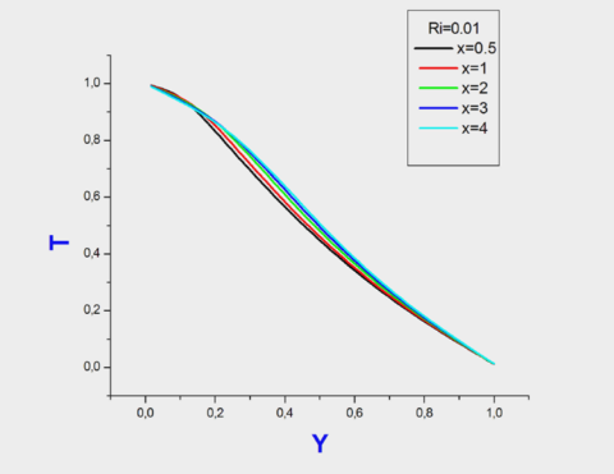


Fig. 4 Variation of $\frac{1}{2}CfRe^{\frac{1}{2}}$ according to Ri

Fig. 5 shows the temperature profiles for various Richardson numbers and in different stations of the longitudinal direction. It is distinguished that the profiles are similar for all Richardson numbers similar to [3]. The increase of Richardson number accelerates the flow near of the wall that explains the influence of the contribution of the natural convection in flow. However, the mixed convection appears when $Ri \geq 2$ and free convection effects are negligible for $Ri << 1$.



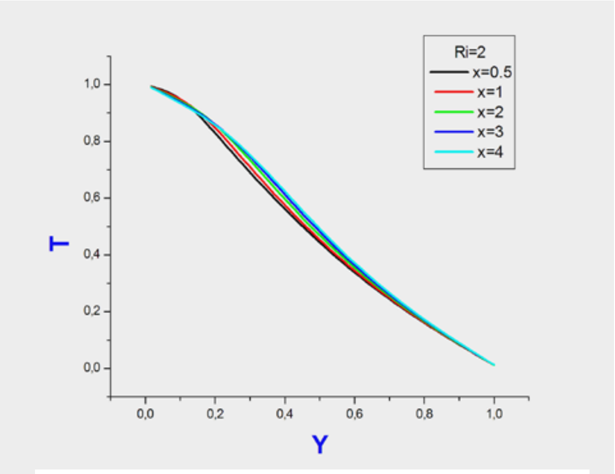


Fig. 5 Temperature profiles for different stations

6. Conclusions

A study of physical parameters effects on the dynamic thermal and mass behavior of the boundary layer was conducted. It was showed that the increase of Richardson number accelerates the flow and favors heat exchanges. Natural convection is generated by a movement of the temperature gradients giving rise to a movement of fluid particles. So heat and mass transfers destabilize the average flow under the buoyancy effect. As results, the natural convection is reduced which causes the fluid flows is very slowly. This velocity reduction causes an increase in dynamic boundary layer thickness. This, in turn, decreases the rate of heat transfer.

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