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# Entropy Generation Study of Mixed Convection in Porous Channel for Different Prandtl Numbers

Amel Tayari[1], Atef Eljery[1,2], Mourad Magherbi[3] and Ammar Ben Brahim[1]

[1]*Applied Thermodynamics Unit, National School of Engineers of Gabes (ENIG), University of Gabes (UG), Omar Ibn El Khattab Street, 6029 Gabes, TUNISIA*

[2]*Chemical Engineering Department, Faculty of Engineering, King Khaled University (KKU), Asir, Abha, SAUDI ARABIA*

[3] *Civil Engineering Department, High Institute of Applied Sciences and Technology, University of Gabes (UG), Omar Ibn El Khattab Street, 6029 Gabes, TUNISIA*

**Abstract:** This work deals with the numerical study of mixed convection in a saturated porous medium which is enclosed in horizontal channel. A laminar flow model for mixed convection with porous media is the focus of this work. The porous media is modeled through the Brinkman-extended Darcy's equation. The Boussinnesq-Oberbek approximation is used to simulate the effects of mixed convection. The Control Volume Finite Element Method is used to elaborate the computational code. Then, Implicit Alternates Directions method is used for solving the governing equations. The coupled pressure-velocity is treated by using the SIMPLER algorithm. The effect of the Prandtl, the modified Brinkman, the Darcy and the Raleigh numbers on the total entropy generation as well as on averaged Nusselt number are studied.

**Key words:** Numerical method, Entropy generation, porous media, mixed convection, Darcy, Brinkman.

## 1. Introduction

Analysis of a laminar flow in a channel filled with saturated porous media has significantly increased during recent years because the interaction between the clear fluid and the porous system is diverse and complex. A large overview of flow through porous media for many systems and situations are well documented in the literature [1, 2]. Al-Hadhrami et al. [3] investigated the combined free and forced convection of a fully developed Newtonian fluid within a vertical channel composed of porous media when viscous dissipation effects are taken into consideration. Tao [4] investigated the fully developed mixed convection with uniform wall temperature in a vertical channel.

**Corresponding Author:** Atef Eljery
E-mail: Atefeljery@hotmail.fr

The mixed convection in a vertical channel filled by a porous medium is studied by Ingham et al. [5] with viscous heating effect. Umavathi et al. [6, 7] examined numerically and analytically, mixed convection in a vertical channel filled with a porous medium using Brinkman-Forchheimer model. The second law of thermodynamics is applied to investigate the irreversibility in terms of entropy generation. Bejan [8] was the first author that studied the entropy generation in a convective heat transfer in a pipe flow. In all cases considered in his research he found that the pipe wall region acted as a strong concentrator of irreversibility. Baytas [9] investigated the entropy generation for free and forced convection in a porous cavity and a porous channel for different flow regimes. The entropy generation rate in a laminar flow through a channel filled with saturated porous media was investigated by

[10] and [11] for different thermal boundary conditions. The Brinkman model was employed. The result showed that the heat transfer irreversibility dominated over the fluid friction irreversibility and the viscous dissipation had no effect on the entropy generation rate at the centerline of the channel. A numerical study is reported by Hooman et al. [12] to investigate the entropy generation due to forced convection in a parallel plate channel filled by a saturated porous medium. Two different thermal boundary conditions were considered being isoflux and isothermal walls. Increasing the porous media shape factor and the Brinkman number, and decreasing the dimensionless degree of irreversibility of the problem, as reflected in (Ns). Moreover, one concludes that different arrangement of the parameters will lead to completely different behavior for both (Ns) and (Be) as described. Guo et al. [13] numerically studied the effect of viscous dissipation on entropy generation for laminar flow region for different fluids in curved square microchannels. Furthermore, Rajiv Dwivedi [14] presented the application of the second law of thermodynamics to the incompressible viscous laminar flow through a channel filled with porous media. The result shows that the viscous dissipation has no effect on the entropy generation rate at the centerline of the channel. Entropy generation in a vertical square channel packed with saturated porous media, and subjected to differentially heat isothermal walls was numerically investigated by Abdulhassan et al. [15]. He showed that the value of the entropy generation number decreases as the Reynolds number, Darcy number increases and Eckert number decreases. The results indicate that irreversibility due to fluid friction dominate for higher Darcy numbers, while as Darcy decrease, the irreversibility dominates due to the heat transfer.

The aim of this investigation is to study the fully developed mixed convection in a horizontal channel filled by a porous medium under a vertical temperature gradient. The analysis was performed using Darcy–brinkman formulation with the Boussinesq approximation. Influence of Rayleigh number, Prandtl number and the Darcy number on the entropy generation due to heat transfer and viscous friction effect was investigated.

## 2. Mathematical formulation

The system under consideration is a horizontal channel filled with a saturated porous media. To avoid discontinuity, the temperature of incoming stream is assumed to vary linearly from $T_h$ (hot temperature) at the bottom wall to $T_c$ (cold temperature) at the upper wall.The medium is assumed to be isotropic, homogeneous and in thermodynamic equilibrium with the fluid. The flow in the porous channel is laminar and two-dimensional. All physical properties of the fluid are assumed to be constant, except its density which satisfies the Boussinesq approximation such that:

$$\rho = \rho_0 \left[ 1 - \beta_T (T - T_0) \right] \tag{1}$$

$\rho_0$ , $T_0$ and $\beta_T$ are the density of fluid, the reference temperature and the thermal volumetric expansion coefficients, respectively. It is given by:

$$\beta_T = \frac{1}{\rho_0} \left( \frac{\partial p}{\partial T} \right)_p \tag{2}$$

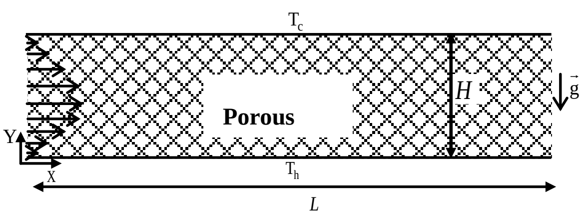


**Figure1. Schematic diagram of the problem.**

By using the following dimensionless variables:

$$\tau = \frac{t}{H} u_0 \, ; X = \frac{x}{H} \, ; Y = \frac{y}{H} \; ; \quad V_x = \frac{v_x}{u_0} \, ; V_y = \frac{v_y}{u_0} \, ; P = \frac{p}{\rho_0 u_0^{\,2}} \, ; \theta = \frac{T - T_c}{T_h - T_c} \tag{3}$$

Using the Darcy-Brinkman model, the dimensionless equations are written as follows:

$$div(V) = 0 \tag{4}$$

$$\frac{\partial V_x}{\partial \tau} + div\left(J_{V_x}\right) = -\varepsilon \frac{\partial P}{\partial X} - \frac{\varepsilon}{Da.\mathrm{Re}} V_x \tag{5}$$

$$\frac{\partial V_y}{\partial \tau} + div\left(J_{V_y}\right) = -\varepsilon \frac{\partial P}{\partial Y} - \frac{\varepsilon}{Da.\mathrm{Re}} V_y + \frac{Ra.\varepsilon}{\mathrm{Re}.Pe} \theta \tag{6}$$

$$\sigma \frac{\partial \theta}{\partial \tau} + div\left(J_\theta\right) = 0 \tag{7}$$

where : $J_{V_x} = \frac{1}{\varepsilon} V_x V - \frac{\Lambda \varepsilon}{\mathrm{Re}} grad\left(V_x\right)$

$$J_{V_y} = \frac{1}{\varepsilon} V_y V - \frac{\Lambda \varepsilon}{\mathrm{Re}} grad\left(V_y\right)$$

$$J_\theta = \theta V - \frac{1}{\mathrm{Re}.\mathrm{Pr}} grad\left(\theta\right)$$

The boundary and initial conditions appropriate to laminar flow within the differential heated porous channel are:

$$0 \le X \le L/H \;;\; Y = 0;\; V_x = V_y = 0$$

$$;\theta = 1 \; 0 \le X \le L/H \;;\; Y = 1;\; V_x = V_y = 0 \;;\theta = 0$$

$$X = 0\,; 0 \le Y \le 1\,; V_x = 6Y(1-Y)\,; V_y = 0\,; \theta = 1 - Y \tag{8}$$

$$X = L/H\,; 0 \le Y \le 1\,; \frac{\partial \varphi}{\partial \tau} + \frac{\partial \varphi}{\partial X} = 0, \int_0^1 V_X\, dY = 1, (\varphi = V_X, V_Y)$$

$$\text{At} \;\; \tau = 0;\; V_x = V_y = 0\,; P = 0\,;\; \theta = 0.5 - X$$

## 3. Entropy generation

According to Mahmud and Fraser [16] the local volumetric rate of entropy generation for a viscous incompressible fluid defined by:

$$S_{\mathrm{l,a}} = \left(\frac{\partial \theta}{\partial X}\right)^2 + \left(\frac{\partial \theta}{\partial Y}\right)^2 + \frac{Br^*}{Da}\left(V_x^2 + V_y^2\right) + Br^*\left[2\left(\frac{\partial V_x}{\partial X}\right)^2 + 2\left(\frac{\partial V_y}{\partial Y}\right)^2 + \left(\frac{\partial V_x}{\partial Y} + \frac{\partial V_y}{\partial X}\right)^2\right] \tag{9}$$

The first term on the right-hand side of Eq. 9 represents the heat transfer part of local entropy generation, the second is the Darcy viscous entropy generation and the third represents the clear fluid viscous entropy generation.

Where *Da* and $\mathrm{Br}^*$ are the Darcy number and the modified Brinkman number respectively. The dimensionless total entropy generation for the entire channel is obtained by integrating (9):

$$S_{\mathrm{t}} = \int_0^1 \int_0^{L/H} S_{\mathrm{l,a}}\, dxdy \tag{10}$$

From the expression for total entropy generation number (10), the time-averaged total entropy generation can be evaluated using the following equation:

$$\langle \mathrm{S_t} \rangle = \frac{1}{\Theta} \int_0^{\Theta} \mathrm{S_t}\, d\tau \tag{11}$$

The thermal heat flux exchanged between the walls and the flow is characterized by the space-averaged Nusselt number evaluated as follows:

$$\langle Nu \rangle = \frac{1}{L/H} \int_0^{L/H} Nu dX \tag{12}$$

where Nu is the local Nusselt number defined as:

$$Nu = \left|\frac{\partial \theta}{\partial Y}\right| \tag{13}$$

## 4. Numerical procedure

The purpose of using the numerical method is the determination of the temperature and the velocity scalar fields. From the known temperature and velocity fields, calculated at any time local entropy generation $\mathrm{S_{l,a}}$ is then obtained. The total entropy generation is calculated by numerical integration. The numerical used method consists on the Control Volume Finit Element Method (CVFEM) of Saabas and Baliga [17]. The used numerical code written in FORTRAN language was described and validated in details in Abbassi et al. [18, 19].

In order to assess the accuracy of our numerical technique, the results obtained by the present method are compared with those of the laminar flow in a horizontal porous channel reported by Mahmud and

Fraser [16] and the laminar flow in a vertical porous channel given by Abdulhassan et al. [15]. A good agreement between our results and the previous ones as illustrated in table 1.

**Table 1: Maximum dimensionless velocity component in X direction for Pr=0.7, Re=100.**

| Darcy number | This study | Mahmud and Fraser [16] | Abdulhassan et al. [15] |
|---|---|---|---|
| 1.0e-9 | 1.02 | 1.01 | - |
| 0.001 | 1.06 | 1.06 | 1.09 |
| 0.01 | 1.23 | 1.11 | 1.30 |
| 0.05 | 1.40 | 1.26 | - |
| 0.1 | 1.44 | 1.33 | 1.55 |
| 1 | 1.53 | 1.48 | 1.59 |
| 10 | 1.57 | 1.50 | 1.59 |
| 100 | 1.57 | 1.50 | 1.59 |
| 1000 | 1.57 | 1.50 | 1.59 |

## 5. Results and discussions

In this paper, the porosity and the Reynolds numbers are fixed at 0.5. The operating parameters are in the following ranges: $10^3 \leq Ra \leq 10^5$; $10^{-6} \leq Da \leq 10$; $10^{-5} \leq Br^* \leq 10^{-1}$; $4 \leq Pr \leq 0.2$. The viscosity ratio and the specific heat capacity ratio they are fixed to unity.

The variation of total dimensionless entropy generation versus dimensionless time for different Prandtl number is illustrated in Fig. 2. The Raleigh, the Reynolds, the Darcy and the porosity numbers are fixed at $10^4$, 10, $10^{-2}$ and 0.5 respectively. As can be seen from the indicated figure the evolution of entropy generation in time is asymptotic for Pr = 4 and oscillatory asymptotic for Pr = 1.5. Oscillatory periodic behaviour of entropy generation is observed for Prandtl ranges between 1 and 0.2. This behaviour indicates a periodic structure inside the system. This structure maintained by the consumption of an energetic portion received by the system is known as dissipative structure. As a consequence, the system acts in the non linear branch of thermodynamics of irreversible processes. From this figure, one can see that value of entropy generation increases with the decreasing of Prandtl number. This is due to the augmentation of irreversibility due to viscous effects. The irreversibility due to the heat transfer is practically non effect on entropy generation for low Prandtl numbers.

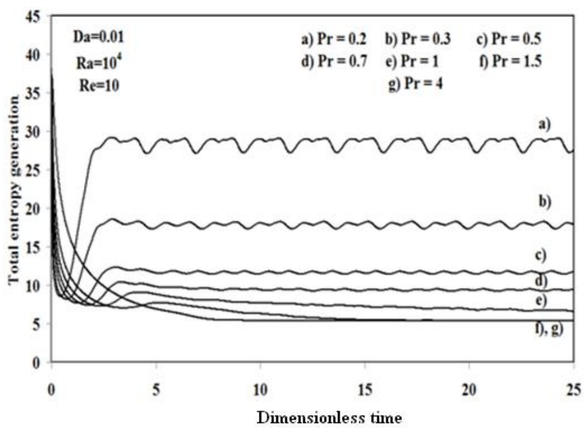


**Fig. 2 Variation of total dimensionless entropy generation versus dimensionless time for different Prandtl number at Ra = $10^4$, Re = 10, Da = $10^{-2}$ and ε = 0.5.**

Figure 3 illustrates the evolution of the total entropy generation with the Darcy number for different values of Prandtl number. The Raleigh, the Reynolds and the porosity numbers are fixed at $10^4$, 10 and 0.5

respectively. Because of the important difference between the entropy generation values, alogarithmic scale is used. As can be seen from Fig. 3, for low values of Darcy number (less than $10^{-3}$), the time-avearge entropy generation decreases rapidly when increasing the Darcy number. In this case, the Prandtl number has not effect on the total entropy generation. This decreasing of the total entropy generation can be justified by noting that, for very small Darcy number, the velocity and temperature gradients are insignificant, and consequently the clear fluid viscous and the heat transfer dissipations come to be negligible. Only the Darcy viscous entropy generation ($S_{l,a,D}$) persists, and presents important values for very small Darcy number. When the Darcy number increase (higher than $10^{-3}$), total entropy generation slightly increase. For Darcy number equal to 1, the total entropy generation increases when the Prandtl number decreases. This increase due to the predominance of the clear fluid viscous and the heat transfer irreversibility compared to Darcy viscous irreversibility.

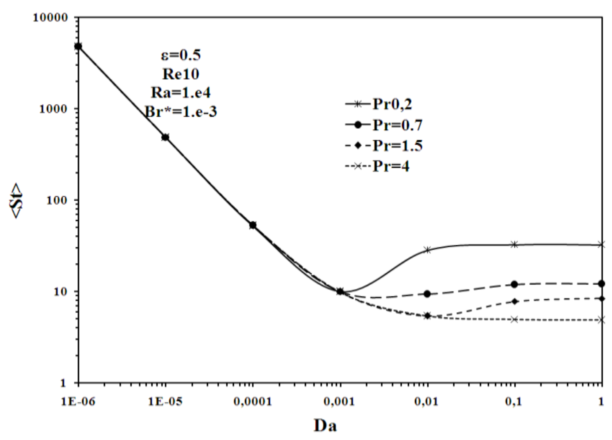


**Fig. 3 Variation of average entropy generation as function a Darcy number for different values of Prandtl number at ε = 0.5, Re = 10 and Br* = $10^{-3}$.**

The variation of the time-average entropy generation as a function of modified Brinkman number is plotted in Fig. 4 for different value of Prandtl number. The Raleigh, Darcy and porosity numbers are fixed at $10^4$, $10^{-2}$ and 0.5 respectively. It can be concluded that for a given modified Brinkman number value, total entropy generation decreases with the Prandtl number. For a fixed Prandtl number, the total entropy generation is an increasing function of the modified Brinkman number. Furthermore, Fig. 4 shows a slight increase of the total entropy generation with the Prandtl number at small values of modified Brinkman number and a rapid increase of the total entropy generation at relatively high value of the modified Brinkman number. This means that the irreversibility due to viscous effects is the dominant part of the total entropy generation ($S_{l,a}$) over that the irreversibility due to the heat transfer.

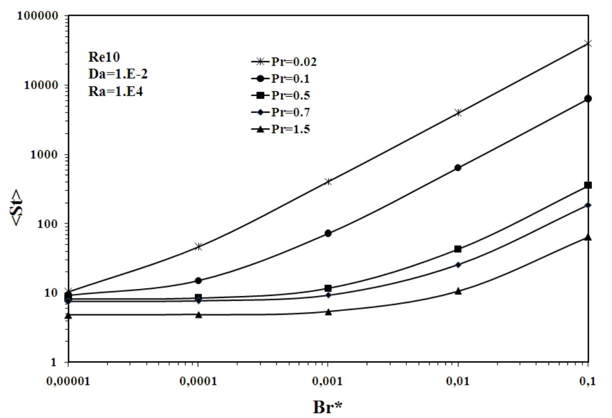


**Fig. 4 Variation of the averaged entropy generation as a function of modified Brinkman number for different Prandtl number at Re = 10, Ra = $10^4$ and Da = $10^{-2}$.**

Figure 5 show the effect of the Darcy number on the heat transfer for different values of Raleigh number, varied in the range $10^3$ to $10^5$. The Prandtl and the porosity numbers are fixed at 0.7 and 0.5 respectively. The modified Brinkman number fixed at $10^{-3}$. The alogarithmic scale is used in the bottom axis. As can be seen from this figure, at low value of Rayleigh numbers (Ra = $10^3$) the Darcy number has not effect on the averaged Nusselt number. For low values of Darcy number ($Da$=$10^{-4}$), the variation of averaged Nusselt number is practically constant. The convection is insignificant, thus the flow is converted into conduction regime. For high Darcy number the averaged Nusselt number increases with Rayleigh number. The convective heat transfer effect is predominant. For a fixed value of Rayleigh number, the effect of the Darcy number is more and more pronounced. Thus, the velocity and thermal gradients increase inducing an increase of the thermal and viscous dissipation and consequently an increase of the averaged Nusselt number.

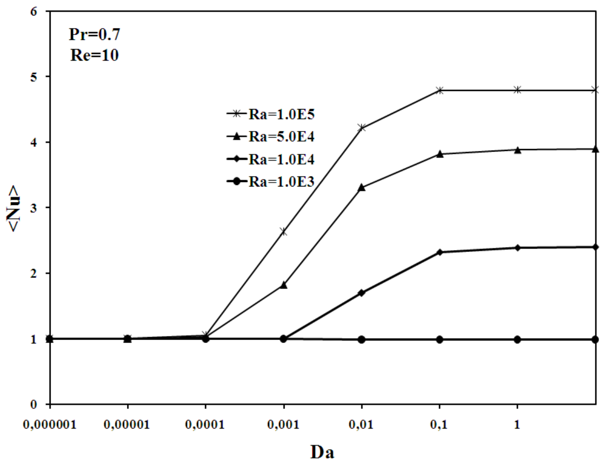


**Fig. 5 Influence of the Darcy number on Nusselt number for different values of Raleigh number at ε = 0.5, Pr = 0.7, Re = 10 and Br* = $10^{-3}$.**

## 6. Conclusion

In this work, the Navier–Stokes and energy equations are modeled by Darcy-Brinkman model. Influence of dimensionless parameters on entropy generation in mixed convection through a porous channel is numerically studied at fixed value of porosity at ε = 0.5. The most important notice points given by the present investigation are the following:

1- For Darcy, Rayleigh and Brinkman numbers fixed at Da = $10^{-2}$, Ra = $10^{4}$ and Br$^{*}$ = $10^{-3}$ respectively. Results show the entropy generation increases with the decreasing of Prandtl number. This is due to the augmentation of irreversibility due to viscous effects.
2- The total entropy generation decreases rapidly when increasing Darcy number from $10^{-6}$ to $10^{-3}$. This case corresponds to a dominance of Darcy viscous entropy generation. The Prandtl number effect on total entropy generation is well seen for Darcy number upper than $10^{-3}$, therefore the convection effects in the porous channel beginning to be more pronounced. As a consequence, the clear fluid viscous entropy generation increases, whereas the Darcy viscous irreversibility decreases. For decreasing Prandtl number, results show a rapid increase of the total entropy generation at relatively high value of the modified Brinkman number.
3- The Prandtl and the modified Brinkman numbers are fixed at 0.7 and $10^{-3}$ respectively. It can be concluded that the Nusselt number increase with the Darcy number for a fixed value of Raleigh number (≥ $10^{4}$). For Darcy number more than $10^{-4}$, Nusselt number increases.

## Nomenclature

*Da* : Darcy number ($K/H^2$)
K : Permeability of the porous media ($m^2$)
g : gravitational acceleration ($m.s^{-2}$)
Ra: Rayleigh number in porous media ($\beta g \Delta T H^3/\upsilon.\alpha_{eff}$)
Re: Reynolds number ($Hu_0/\upsilon$)
Pe: Peclet number (Re.Pr)
Br:Brinkman number (Ec.Pr)
Br*: modified Darcy-Brinkman number (Br/Ω)
Ec: Eckert number ($u_0^2/c_p\Delta T$)
$u_0$ :average velocity ($m.s^{-1}$)
H: channel width (m)
L: length of the channel (m)
p: pressure nondimensionalized ($N.m^{-2}$)
P: dimensionless pressure
Pr: Prandtl number ($\mu c_p/k_m$)
t: time (s)
T: temperature (K)
$T_0$: mean Temperature [$(T_h+T_c)/2$] (K)
ΔT: temperature difference ($T_h-T_c$)
⟨Nu⟩: the space-averaged Nusselt number
S: dimensionless entropy generation ($J.s^{-1}.K^{-1}$)

$\langle S_t \rangle$: time average entropy generation ($J.s^{-1}.K^{-1}$)
$v$ : dimensional velocity vector ($m.s^{-1}$)
$V$: dimensionless velocity vector
$v_x, v_y$: velocity components in x and y directions respectively ($m.s^{-1}$)
$V_x, V_y$: Dimensionless velocity components in X and Y directions respectively
$x,y$: Cartesian coordinates (m)
$X,Y$ : dimensionless Cartesian coordinates

**Greek symbols**

β: Thermal expansion coefficient ($K^{-1}$)
ε : porosity of the porous medium
θ: dimensionless temperature
Θ : dimensionless period
ρ : mass density ($kg.m^{-3}$)
σ: specific heat capacities ratio ($(\rho c)_m/(\rho c)_f$)
Λ: viscosity ratio ($\mu_{eff}/\mu$)
μ: dynamic viscosity ($kg.m^{-1}s^{-1}$)
υ : kinematic viscosity ($m^2.s^{-1}$)
τ : dimensionless time
Ω : dimensionless temperature difference ($\Delta T/T_0$)

**Subscripts**

*a* : dimensionless
*c:* cold wall
*F:* fluid friction
*H:* heat transfer
h : hot wall
l : local
t : total
m : porous media
f : fluid
s : solid

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