International Journal of Mechanics and Energy (IJME)
Vol. 2, Issue 3, 2014, ISSN: 2286-5845

# Temperature-dependent viscosity effect on free convection in a square cavity filled with a shear-thinning and subjected to cross thermal gradients

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**Abstract:** Study of buoyancy driven convection of thermo-dependent a shear-thinning power-law fluid confined in a square cavity, submitted to cross uniform heat fluxes is conducted numerically using a finite difference method. The combined effects of the ratio between the cross heat fluxes and the thermo-dependency parameter on the flow and thermal fields, and the resulting heat transfer are examined and discussed.

**Key words:** Heat transfer; Natural convection; Non-Newtonian fluids; Numerical study; Square cavity.

## 1. Introduction

Thermal buoyancy convection is a flow resulting from density variations within a non-isothermal fluid under the gravity effect. Such a phenomenon is of importance in various domains, which attracted many worldwide researchers, through the decades, to investigate it in many geometrical configurations and under various boundary conditions. Useful literature review can be found in the book by Gebhart et al. [1], respectively, where most of the fluids considered are of Newtonian behavior.

On the other hand, given the obvious relevance to various manufacturing and processing industries dealing with industrial applications, such as papermaking, oil drilling, slurry transporting, food processing, polymer engineering and so on, the studies number reported on natural convection involving non-Newtonian fluids has been increased during the last two decades [2], but owing to their complex rheological behavior and their particular isothermal or non-isothermal flow conditions more investigations are to be undertaken in this area.

Another challenging problem is the dependence of the rheological properties of these fluids on the temperature. To our best knowledge, most of the reported studies on natural convection in non-Newtonian fluids ignore such an aspect. This can be a serious assumption, since in many cases this effect has a significant influence on heat transfer [3]. Therefore, the goal of the present study is to contribute to a better understanding of the thermo-dependence effects on buoyancy convection heat transfer in such media.

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## 2. Mathematical formulation and solution procedure

### *2.1 Problem statement and viscosity model*

The geometry under consideration is sketched in Fig. 1 It consists of a two-dimensional square enclosure of size $H' \times H'$ subjected to cross uniform densities of heat flux, q' and bq'.

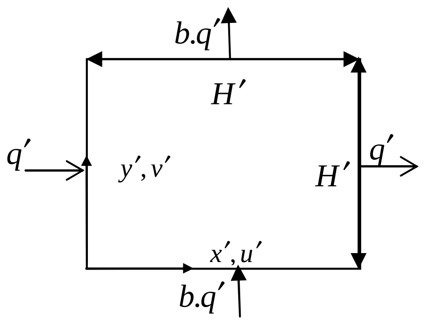


**Fig. 1. Sketch of the geometry and coordinates system**

The non-Newtonian fluids considered here are those whose rheological behaviors can be approached by the power-law model, due to Ostwald-de Waele, which, in terms of laminar effective viscosity, can be written as follows:

$$\mu'_a = k_T \left( 2\left( \left(\frac{\partial u'}{\partial x'}\right)^2 + \left(\frac{\partial v'}{\partial y'}\right)^2 \right) + \left(\frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'}\right)^2 \right)^{\frac{n_T - 1}{2}} \quad (1)$$

The two empirical parameters $n_T$ and $k_T$, appearing in Eq. (1), are the flow behavior and consistency indices, respectively. They are, in general, functions of the temperature, but in most of cases the temperature-dependence of $n_T$ can be ignored $(n_T = n)$ since it is weak compared to that of $k_T$ [3,4], which is described by the Frank-Kamenetski exponential law [5]:

$$k_T = k e^{-c_1 (T' - T'_r)} \quad (2)$$

reflecting the viscosity diminution with the temperature, where $c_1$ is an exponent related to the flow energy activation and the universal gas constant, and $T'_r$ is a reference temperature.

Note that for $n = 1$ the behavior is Newtonian and the consistency is just the viscosity. For $0 < n < 1$, the effective viscosity decreases with the amount of deformation and the behavior is shear-thinning. Conversely, for $n > 1$, the viscosity increases with the amount of shearing, which implies that, the fluid behavior is shear-thickening.

### *2.2 Governing equations and boundary conditions.*

On the basis of the assumptions commonly adopted in natural convection problems and using the characteristic scales $H'$, $H'^2/\alpha$, $\alpha/H'$, $\alpha/H'^2$, $q'H'/\lambda$ and α, which correspond respectively to length, time, velocity, vorticity, temperature and stream function, the dimensionless governing equations for Boussinsq-temperature-dependent viscosity fluids, written in terms of vorticity, Ω, temperature, T, and stream function, ψ, are as follows:

$$\frac{\partial \Omega}{\partial t} + \frac{\partial (u\Omega)}{\partial x} + \frac{\partial (v\Omega)}{\partial y} = \Pr\left[ \mu_a \left[ \frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2} \right] + 2\left[ \frac{\partial \mu_a}{\partial x}\frac{\partial \Omega}{\partial x} + \frac{\partial \mu_a}{\partial y}\frac{\partial \Omega}{\partial y} \right] \right] + S_\Omega \quad (3)$$

$$\frac{\partial T}{\partial t} + \frac{\partial (uT)}{\partial x} + \frac{\partial (vT)}{\partial y} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \quad (4)$$

and

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\Omega \quad (5)$$

where

$$u = \frac{\partial \psi}{\partial y};\ v = -\frac{\partial \psi}{\partial x};\ \Omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad (6)$$

$$\mu_a = e^{-mT} \left[ 2\left[ \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 \right] + \left[ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right]^2 \right]^{\frac{n-1}{2}} \quad (7)$$

and

$$S_{\Omega} = \Pr\left[\left[\frac{\partial^2 \mu_a}{\partial x^2} - \frac{\partial^2 \mu_a}{\partial y^2}\right]\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right] - 2\frac{\partial^2 \mu_a}{\partial x \partial y}\left[\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right]\right] + \Pr Ra \frac{\partial T}{\partial x} \quad (8)$$

For the present problem, the appropriate non-dimensional boundary conditions are:

$$u = v = \psi = \frac{\partial T}{\partial x} + 1 = 0 \quad \text{for } x = 0 \text{ and } 1 \quad (9)$$

$$u = v = \psi = \frac{\partial T}{\partial y} + b = 0 \quad \text{for } y = 0 \text{ and } 1 \quad (10)$$

Note that the major disadvantage of this formulation lies in the fact that Ω is unknown at the boundaries. To overcome such a difficulty, the Woods formulation has been adopted for stability and accuracy reasons [6].

*2.3 Governing parameters*

In addition to the flow behaviour index, *n*, three other dimensionless parameters appear in the governing equations. These are the Pearson, Prandtl and Rayleigh numbers defined, respectively, as:

$$m = -\frac{1}{k_T}\frac{dk_T}{\mathrm{dT}} = -\frac{d \ln(k_T/k)}{\mathrm{dT}},$$

$$\Pr = \frac{(k/\rho)\, H'^{2-2n}}{\alpha^{2-n}} \quad (11)$$

$$\text{and} \quad Ra = \frac{g\, \beta\, H'^{2n+2} q'}{(k/\rho)\alpha^n\, \lambda}$$

The Pearson number [7], which is a new dimensionless quantity taking place in this study, measures the effect of temperature change on the effective viscosity.

*2.4 Heat transfer*

The steady solution has been used to calculate the average Nusselt number in the horizontal and vertical directions, respectively, defined as:

$$Nu_h = \frac{q'H'}{\lambda \overline{\Delta T'_v}} = \frac{1}{\overline{\Delta T_v}} \quad (12)$$

$$Nu_v = \frac{bq'H'}{\lambda \overline{\Delta T'_h}} = \frac{b}{\overline{\Delta T_h}} \quad (13)$$

where $\overline{\Delta T_v}$ is the average temperature difference between the two vertical walls and $\overline{\Delta T_h}$ is the average temperature difference between the two horizontal walls.

*2.5 Heatlines formulation*

The visualization of the paths followed by the heat flow through the enclosure requires the use of the heatlines concept, which consists of lines of constant heat function, *H*, that are defined, according to Kimura and Bejan [8], from the following equations

$$\frac{\partial H}{\partial y} = uT - \frac{\partial T}{\partial x}; \quad -\frac{\partial H}{\partial x} = vT - \frac{\partial T}{\partial y} \quad (14)$$

whose derivation, with respect to *x* and *y*, and combination give rise to

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = -\frac{\partial vT}{\partial x} + \frac{\partial uT}{\partial y} \quad (15)$$

To obtain the boundary conditions associated with Eq. (15), an integration of Eq. (14), along the four cavity walls, is necessary, which gives:

$$H(0, y) = H(0,0) \quad \text{for } \mathrm{x} = 0 \quad (16)$$

$$H(\mathrm{x},1) = H(0,1) - \mathrm{x} \quad \text{for } \mathrm{y} = 1 \quad (17)$$

$$H(1, y) = H(1,1) \quad \text{for } \mathrm{x} = 1 \quad (18)$$

$$H(\mathrm{x},0) = H(1,0) + 1 - \mathrm{x} \quad \text{for } \mathrm{y} = 0 \quad (19)$$

Finally, the solution of Eq. (15) yields the values of *H*, in the computational domain, whose contour plots provide the heatline patterns. Note that only the differences between the values of *H* are required instead of its intrinsic ones, which offers the possibility to choose $H(0,0) = 0$ as an arbitrary reference value for *H*.

*2.6 Solution procedure*

The two-dimensional governing equations have been discretized using the second order central finite difference methodology with a regular mesh size. The integration of equations (3) and (4) has been performed with the Alternating Direction Implicit method (ADI), originally used for Newtonian fluids and successfully experimented for non-Newtonian power-law fluids [9-11]. To satisfy the mass conservation, Eq. (5) has been solved by a Point Successive Over Relaxation method (PSOR) with an optimum relaxation factor calculated by the Frankel formula [6]. A grid of 81×81 has been required for obtaining adequate results. At each time step, δt, which has been chosen between $10^{-7}$ and $10^{-4}$ (depending on the values of the parameters b and m), the convergence criterion $\sum_{i,j}\left|\psi_{i,j}^{k+1}-\psi_{i,j}^{k}\right| \Big/ \sum_{i,j}\left|\psi_{i,j}^{k+1}\right| < 10^{-4}$ has been satisfied for $\psi$, where $\psi_{i,j}^{k}$ is the value of the stream function at the node $(i, j)$ for the k[th] iteration level.

### 3. Results and discussion

As was reported in the past by [11], the convection is rather insensitive to Pr variations, provided that this parameter is large enough as it is the case for the non-Newtonian fluids and for a large category of fluids having a Newtonian behavior. Therefore, Pr is not considered as an influencing parameter in this study and the simulations are conducted with $P \to \infty$, i.e. by neglecting the inertia terms on the left hand side of Eq. (3) owing to their negligible contribution. To examine the cross fluxes and the thermo-dependency effects, some results, corresponding to $b$ = 0, 0.2, 0.5, 0.7 and 1, $m$ = 0 and $m$ = 10, $n$ = 0.6 and $Ra$ = 5×10³, are presented and discussed.

Hence, as can be seen from Fig. 2, displaying streamlines (left), isotherms (middle) and heatlines (right), the flow is, in general, unicellular and clockwise, but loses its symmetry with an increasing $m$ for all the values of $b$. Also, the streamlines become more crowded in the region neighboring the left upper corner, which means that the flow is intensified as a result of the viscosity decrease in such a region, giving rise to a stagnation zone which tends to be reduced near the right lower corner and to be extended next to the upper one with an increasing $b$, while for $m$ = 0, the effect of $b$ is such that the streamlines become almost parallel to the central part of each wall.

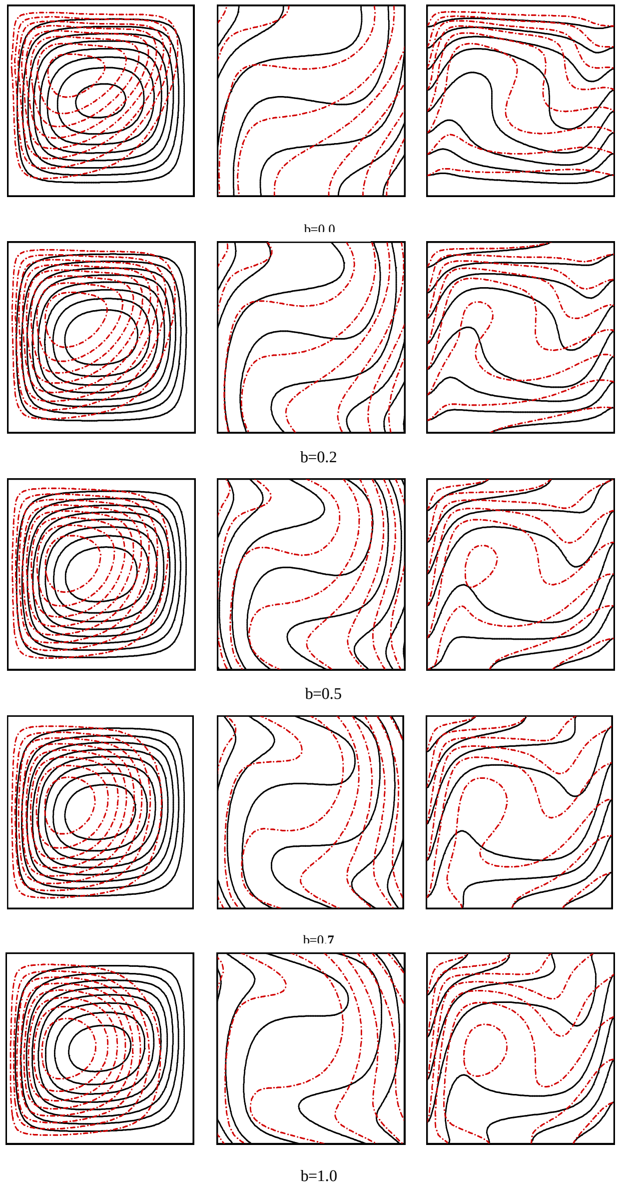


**Fig. 2. Streamlines (left), isotherms (medium) and heatlines(right ) for $Ra = 5.10^3$, $n = 0.6$ and $m = 0$ (black solid line), $m = 10$ (red dashdot line) and various value of b.**

As for the isotherms, they seem to be closely spaced and less distorted in the stagnation region when $m$ passes from 0 to 10, depending on $b$, whose increase leads to their rotation in the counter-clockwise direction.

On the other hand, in order to have a microscopic description of the heat transfer process, which is different from the conventional Nusselt number that describes macroscopically such a phenomenon, a heatlines analysis is required. Hence, with comparison to the iso-consistent case ( $m = 0$ ), the heatlines corresponding to the case $m = 10$ present more distortion, which indicates that the path followed by the heat flow to reach the cold wall is more complicated in the rheological sub-layer. Therefore, the heat transfer is expected to be deteriorated in such a situation. Like the isotherms, an increase of $b$ leads to a deviation of the heatlines in the counterclockwise direction whatever the value of $m$.

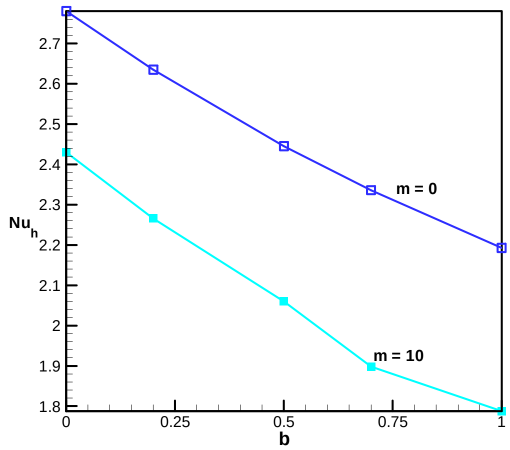


**Fig. 3. Evolution of the average horizontal Nusselt number with b, for Ra = $5.10^3$, n = 0.6, m = 0 and m = 10.**

Moreover, Fig. 3, in which are depicted the variations of $Nu_h$ , shows that this quantity decrease with $b$ whatever the value of $m$. In addition, for a given value of $b$, the same figure indicates a degradation of heat transfer in the horizontal direction with $m$. On the other hand, it can be seen, from Fig. 4, that there exists a critical value of $b$ ($b_c$) corresponding to infinite value of $Nu_v\,(\overline{\Delta T_h} = 0)$ and around which the sign of $Nu_v$ changes. An increase of $m$ anticipates $b_c$.

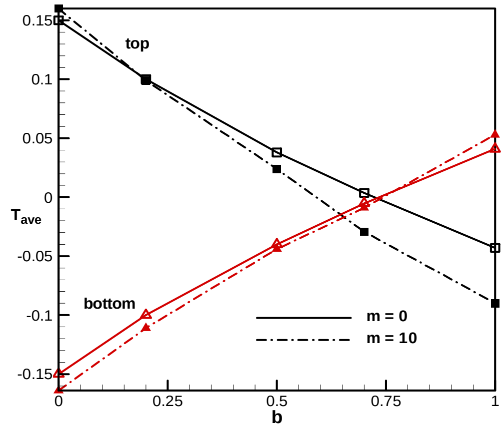


**Fig. 4. Evolution of the average temperature of horizontal walls with b, for $Ra = 5.10^3$ , $n = 0.6$ , $m = 0$ and $m = 10$ .**

## 4. Conclusion

A numerical investigation of steady thermal convection in a square enclosure, filled with shear-thinning power-law fluids and submitted to cross uniform heat fluxes, is performed. The exponential model, due to Frank-Kamenetski, for the viscosity variation with the temperature, is used. The study is focused particularly on combined effects of the ratio between the cross heat fluxes and the thermo-dependency parameter on the flow and thermal fields, and the resulting heat transfer. It emerges that the thermo-dependent behavior affects natural convection heat transfer depending on the proportion of the cross heat fluxes.

## Références

[1] B. Gebhart, Y. Jaluria, R. L. Mahajan, and B. Sammakia, Buoyancy-induced flows and transport, chap. 16, Hemisphere, Washington, DC, (1988).

[2] Mi. Ohta, Ma. Ohta, M. Akiyoshi and E. Obata, A numerical study on natural convective heat transfer of pseudoplastic fluids in a square cavity, Num. Heat Transfer Part A, 41, (2002), 357-372.

[3] C. Nouar, Thermal convection for a thermo-dependent yield stress fluid in an axisymmetric horizontal duct, Int. J. heat Mass Transfer, 48, (2005), 5520-5535.

[4] V. Scirocco, R. Devienne, and M. Lebouché, Ecoulement laminaire et transfert de chaleur pour un fluide pseudo-plastique dans la zone d'entrée d'un tube, Int. J. Heat Mass Transfer, 28(1), (1985), 91-99.

[5] V.S. Solomatov, and A.C. Barr, Onset of convection in fluids with strongly temperature-dependent power-law viscosity, Physics of the Earth and Planetary Interiors, 155, (2006), 140-145.

[6] P.J. Roache, Computational Fluid Dynamics, New Mexico: Hermosa Publishers, (1982).

[7] C. Métivier, C. Nouar, Linear stability of the Rayleigh–Bénard Poiseuille flow for thermodependent viscoplastic fluids, Journal of Non-Newtonian Fluid Mechanics, 163, (2009), 1-8.

[8] S. Kimura and A. Bejan, The Heatline Visualization of Convective Heat Transfer, ASME Journal of Heat Transfer, 105, (1983), 916–919.

[9] S Turki, Contribution to Numerical Study of Natural and Mixed Convection Heat Transfers in Confined Non-Newtonian Fluids, Ph.D. Thesis, CNAM, Paris, France, (1990).

[10] H. Ozoe, and S.W. Churchill, Hydrodynamic stability and natural convection in Ostwald–De Waele and Ellis fluids: the development of a numerical solution, AIChE J., 18, (1972), 1196-1207.

[11] M. Lamsaadi, M. Naïmi, and M. Hasnaoui, Natural convection of non-Newtonian power-law fluids in a shallow horizontal rectangular enclosure uniformly heated from the side, Energy Conversion and Management, 47, (2006), 2535-2551.