

Response of a Plane Free Jet Subjected to Sinusoidal Excitations

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Abstract: A numerical study of the instabilities is realized for 2D incompressible, isothermal and plane jet. The eulerian numerical finite volume method is used. Numerical parameters and boundary conditions are optimized for the studied configuration. This study was operated for moderate Reynolds numbers. The flow is perturbed at the entry of the nozzle. The excitation frequency is a harmonic or sub-harmonic of the natural instability. The instability amplification depends on the excitation frequency. A response mode inherent to the shear flows was detected. The vortex energy is amplified allowing to know their sinuous or varicose behavior. The vortex dissociation and pairing phenomena are highlighted.

Key words: jet, vortex, instability, excitation, frequency, mode.

1. Introduction

The study focuses on the instabilities development in a two-dimensional, isothermal and incompressible jet flow. The eulerian numerical finite volume method is used. The range of moderate Reynolds numbers between 100 and 1000. A simulation series is performed in order to characterize the vortex structures and their development. The disturbance technique is frequently used to highlight the instability mechanism. A sinusoidal excitation is imposed at the inlet of the jet. The response modes are thus determined and the phenomena of detachment and vortex pairing are highlighted. This study seeks to compare our results with those obtained by Ho and Huang [1] for the case of a mixing layer. A relationship was established between the excitation frequency and the response one of the flow. The consequences of this excitation on the evolution of vortex structures are determined.

Hussain and Thompson [2] have demonstrated that the imposed excitations have a limited impact on

average velocities but act on the fluctuations. They distinguish zones in terms of the excitation frequency. These authors show that there is no effect on the flow when the forcing frequency is more than twice greater than or lower than the natural frequency. The effect is to stabilize the frequency in this interaction range mainly for multiples or sub-multiples of this natural frequency. These properties will be discussed in the present paper.

2. Geometry and numerical parameters

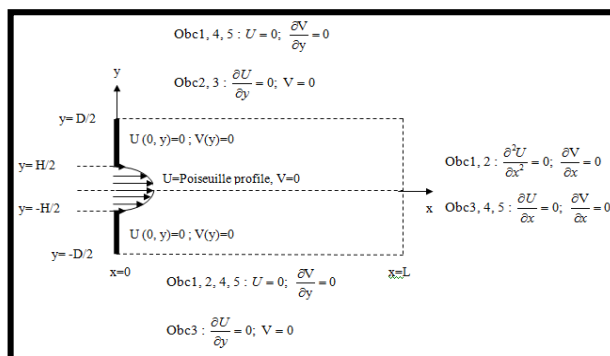


Fig. 1. Studied configuration and boundary conditions.

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The domain of study is the plane free jet. His dimensions are $L \times D = 20 \times 6$. The total length L of the domain is in the longitudinal direction. The height of the nozzle at the entry is $H = 1$ (Fig. 1). The flow is incompressible isothermal and considered as two-dimensional. This corresponds to the conditions that have been found by Faghani *et al.* [3] and Meyer *et al.* [4] in an experimental study at moderate Reynolds numbers. The range of moderate Reynolds numbers has been probed ($100 \leq Re \leq 1000$). Unless explicitly specified, all the numerical runs were performed keeping the same numerical parameters: The mesh size used for most simulations is 400×120 , the time step was chosen 10^{-2} and the test of convergence is ensured for a precision 10^{-7} . To understand the phenomena of vortex pairing, the study at the natural frequency of instability will be processed, then the perturbation method is used. A sinusoidal excitation of the following form is imposed at the inlet of the jet:

$$U_0(0, y, t) = U_0(y, 0) [1 + \beta \sin(2\pi f_e t)] \quad (1)$$

where β is the excitation amplitude optimized at the value 0.025 and f_e is the frequency of variable excitation. U_0 is the mean velocity at the inlet or axial velocity:

$$\begin{cases} U(x=0; -\frac{H}{2} < y < \frac{H}{2}) = U_0(1 - (\frac{2y}{H})^2) \\ U(x=0; -\frac{D}{2} \leq y \leq -\frac{H}{2} \text{ et } \frac{D}{2} \leq y \leq \frac{H}{2}, t) = 0 \end{cases}$$

The selected excitation frequency f_e is a multiple or a sub multiple of the natural frequency of instability f_n . The instability amplification depending on the excitement frequency is determined.

3. Initial conditions

Bickley [5] has determined the velocity profile of a jet resulting from an infinitely long channel in a fluid at rest, the expression is:

$$U = U_0 \operatorname{sech}^2\left(\frac{y}{b}\right) \quad (2)$$

where U is the axial velocity and b represents the growth rate. This case has been treated theoretically by Schlichting [6]. Andrade [7] experimentally confirmed this behavior for a jet resulting from a channel of finite dimension. The experimental of Sato [8] are in agreement with the theoretical results for Reynolds numbers less than 100. In his theoretical study Nolle [9] proposes a more realistic family of profiles than the bickley one which the form is:

$$U = U_0 \operatorname{sech}^2\left(\frac{y}{b}\right)^n = \frac{U_0}{\cosh^2\left(\frac{y}{b}\right)^n} \quad (3)$$

where n is an entire.

These profiles are particularly large near the jet axis (Fig. 2). The case $n = 1$ corresponds to the Bickley profile (Bickley [5]). For $n \geq 2$ the shape of the velocity tends to the uniform square case.

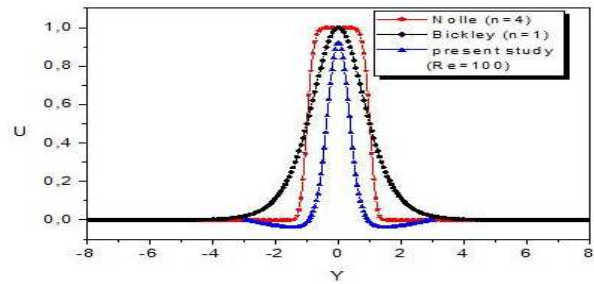


Fig. 2. Comparison of velocity profiles at the inlet

Three initial conditions for the streamwise velocity are tested to validate the numerical simulations corresponding to uniform profile, Poiseuille profile type and profile form ramp (Fig. 3). These velocity profiles are imposed from the input to the output of the jet and for all the height of the nozzle. In the rest of the domain, the velocity is zero. On the jet axis, the initial longitudinal velocity is equal to the unit. The initial transverse component is considered zero in all the domain. The square initial profile does not allow the jet to ease. The axial velocity in all the jet is practically equal to the input velocity (Fig. 3a). The growth rate of the jet is significant. The velocity near lateral boundaries is not negligible. These behaviors can be explained by the non-physical character of the

initial condition. The initial profiles of Poiseuille or ramp type give very similar effects (Fig. 3b). A decrease of the axial velocity is more marked for the Poiseuille profile, supposed to be the closest to the physical reality. Sato [8] realized an experiment showing that the distribution of the mean velocity in the laminar region of a jet depends on the initial condition.

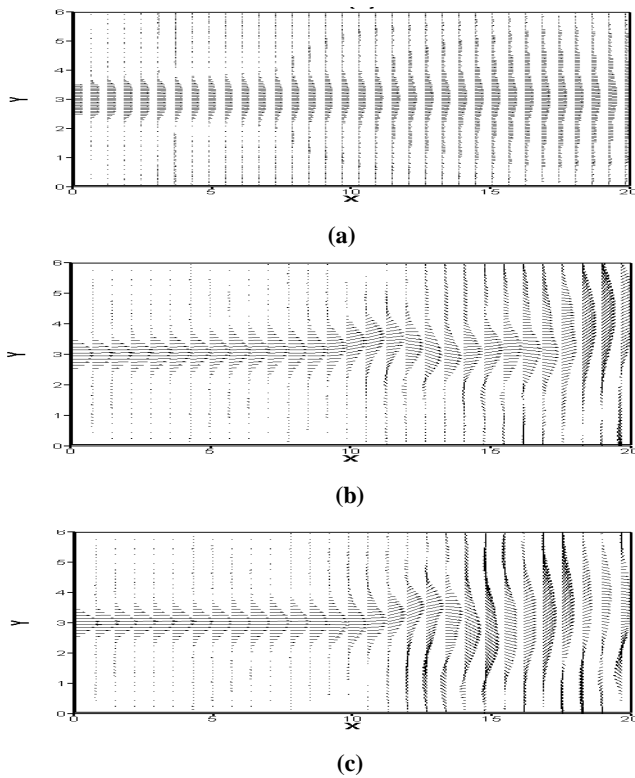


Fig. 3. Velocity vector field for three initial conditions, (Re=500; grid: 400x120; t=120): (a) uniform profile; (b) Poiseuille profile; (c) ramp profile

4. Boundary conditions

The Fig. 4 represents the velocity vector fields for five boundary conditions noted obc1, obc2, obc3, obc4, obc5, indicated in the Fig. 1 for the last time step. The profiles broaden out to reach the lateral boundaries and accurately describe the status of a free jet flow, particularly those related to conditions (obc1). The other conditions affect the flow, and especially perturbations occur at the exit border. The output condition imposed in (obc1) seems to be the

most adequate to the physical problem. It allows the evacuation of the flow. For these reasons, the group of boundary conditions (obc1) was adopted in this study.

These boundary conditions (obc1) are such that: at the entry of the jet U takes the value to the initial condition in the nozzle throughout the simulation, and zero value on the walls. The lateral velocity V is zero on all this border. On the two lateral boundaries, we impose a Dirichlet condition for the longitudinal velocity and a Neumann condition on the transverse velocity. This coupling between the Dirichlet and Neumann conditions allows the training by the lateral borders supposed free. In fact, this boundary which limits the computational domain is not really physical. At the output of the jet Neumann condition is applied to the two velocity components.

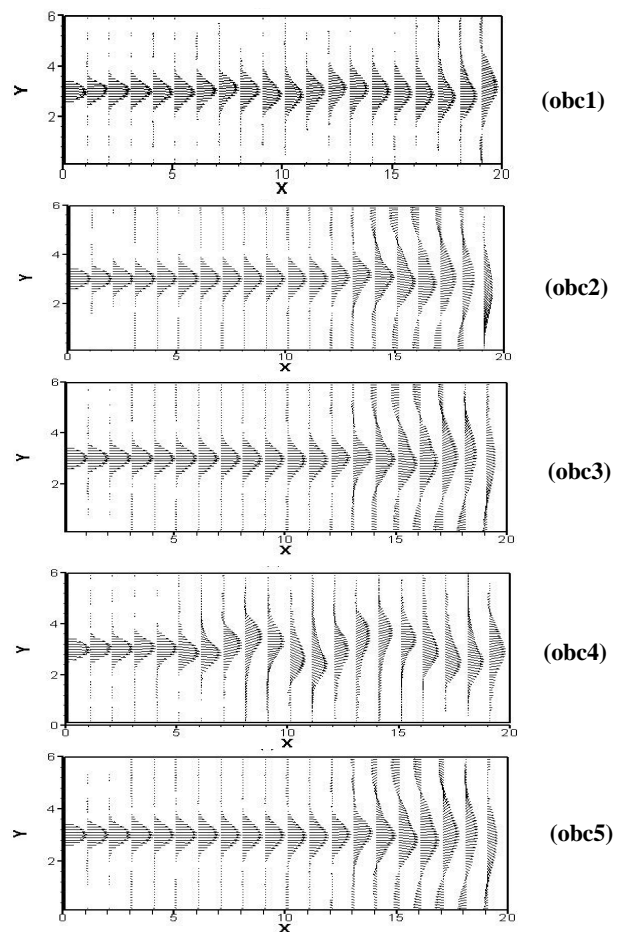


Fig. 4. Velocity vector field for five boundary conditions (Re = 500)

5. Natural jet

5.1 Jet half-width

The half-width of the jet is conventionally defined by $b = y(0.5U_{axe})$. The Fig. 5 show the evolution of the growth rate of the jet b/H dimensionless by the width H of the nozzle, depending on the Reynolds numbers, for different stations of the jet. Near the entry this characteristic varies slightly and remains practically constant. The jet width is not very affected. However, downstream of the flow, it is low for Reynolds numbers less than 300 with a fast decay. The jet takes the same form towards the exit for high Reynolds numbers. Sato [8] showed that the development of the jet width is progressive and becomes directly proportional to the longitudinal distance x .

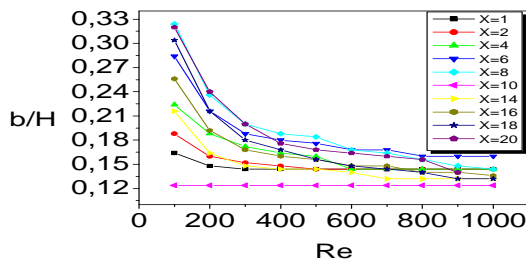


Fig. 5. Natural jet: jet half-width

5.2 Selection of the dominant frequency

The instabilities development is highly related to viscosity. The instability frequency increases with the Reynolds number (Fig. 6). Diffusive effects are considerable for low Reynolds and therefore the instability period is high. These diffusive effects become negligible at high Reynolds and therefore the instability frequency increases.

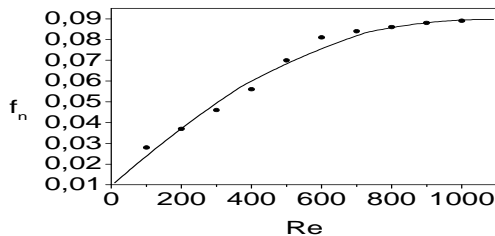


Fig. 6. Natural jet: instability frequency versus Reynolds number

The excitation at the natural frequency ($f_e=f_n$) shows the existence of a preferred mode to the instability wave. The response frequency is equal to the forcing one. The temporal evolution of the velocity has a sinusoidal character at $Re=100$. The signals and the energy spectra are represented in the Fig. 7. The frequency of the most intense peak corresponds to the response frequency $f_r=0.07$. There are other small amplitude peaks in the interaction zone. The excitation effect, particularly at the natural frequency, is to show more clearly the sub-harmonic frequencies i.e to mark clearly the vortex pairing.

6. Selection of response modes

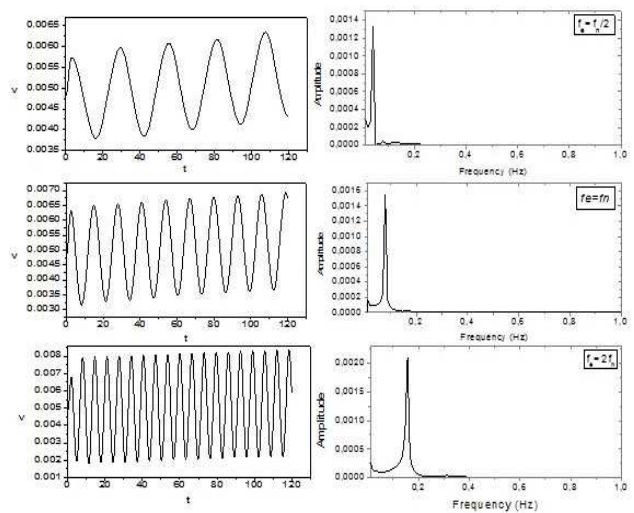


Fig. 7. Temporal evolution of the transverse velocity v and corresponding spectrum for different frequencies, ($x=1$; $y=3$; $Re=100$)

A simulation series is performed in order to highlight the process of response frequency when a monochromatic forcing at multiple or sub-multiples frequencies of f_n is applied. The analysis of the spectrum shows that the response frequency f_r increase with the excitation frequency f_e . Indeed, for $f_e=f_n/2$ the response frequency is 0.03 and for $f_e=f_n$, it is equal to 0.07. This frequency increases to 0.15 for a forcing at $f_e=2f_n$ (Fig. 7).

The Fig. 8 presents the component spectra of the transverse velocity for excitation frequencies smaller

and bigger than f_n . In these spectra, we observe the existence of intense peaks which correspond to the vortex winding and they show the primary Kelvin-Helmholtz instability. This instability propagates downstream and his size becomes larger and ends up by attenuating. The low amplitudes peaks correspond to under harmonics frequencies related to the phenomenon of pairing.

The evolution of the response frequency according to the excitation frequency is represented with a logarithmic scale in the Fig. 9. A preferential amplification of the excitation frequency in the range $[0.15 f_n, 4 f_n]$ is highlighted. In a mixing layer, Ho and Huang [1] and Astruc [10] have shown that the response frequency is equal to the excitation frequency in the interval $[0,5 f_n, 2 f_n]$. Our results complete the results of Sers [11] for an isothermal plane jet to a larger frequency range. This author has shown that the response frequency is closed to the excitation frequency in the range $0.82 < f_e/f_n < 1.13$ for a Reynolds number comparable to the one studied here. Our results also confirm those of Hussain and Thompson [2] stating that the amplitude of the response for excitations at multiples or submultiples of the forcing frequency is weak when the ratio f_e/f_n (or its inverse) overpasses two or three.

The excitation frequency is related to the response frequency following modes. By analyzing the mode diagram in Fig. 9, the relationship between the excitation frequency and the response frequency allows us to understand the phenomenon of vortex fusion. Each mode correspond to a fusion phenomenon of vortex structures. Indeed, the first mode corresponds to the appearance of the first vortex pairing. By propagating downstream, these latter merge and a new modes appear. An energy transfer occurs between the fundamental mode, which is the most amplified one, and his subharmonic. Ho and Huang [1] have shown that the energy transfer translates into a pairing of two successive vortices. The energetic transfers between modes can be

repeated with different sub-harmonics of the natural frequency f_n and lead to successive pairings. For each pairing, the structures size increases almost twice as large in each pairing. The pairings, therefore, contribute, in the vast majority, to the enlargement of the mixed layer.

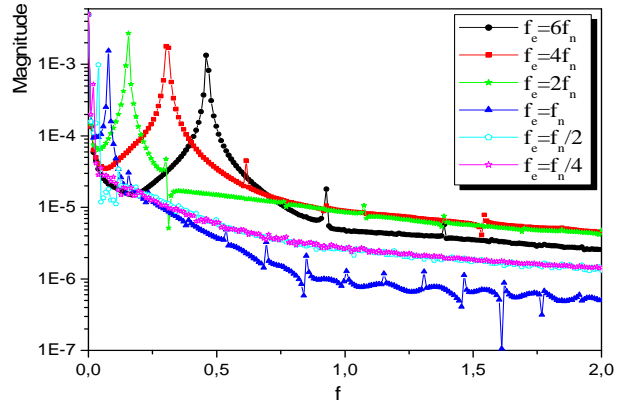


Fig. 8. Spectrum of the transverse velocity component v for different frequencies; $Re=100$

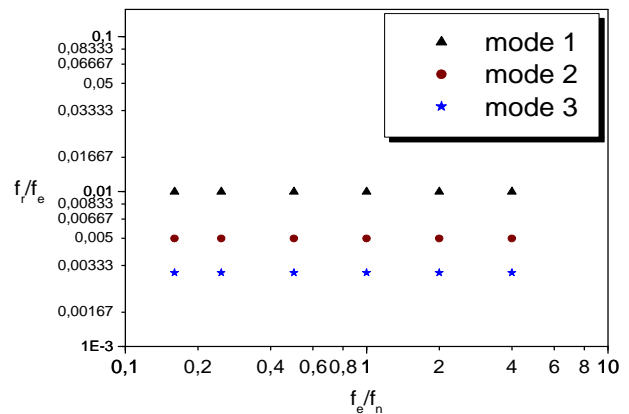


Fig. 9. Response frequency as a function of the excitation frequency

The Fig. 10 presents the iso-vorticities for sub-harmonic and harmonic frequencies of the natural frequency. It is found that the vorticity fields are symmetric with respect to the jet axis over a distance stretching from the input up to $x = 12$ where there is the presence of a varicose mode and the flow maintains a regular form. Near the outlet of the jet, a sinuous mode appears. Vortices have a tendency to dissipate in the flow and they lose their energy. These nonlinear interactions are related to harmonic and

sub-harmonic of the jet dominant frequency. The effect of numerical reflection at the border output is felt; in fact the limit condition cannot translate the physical reality.

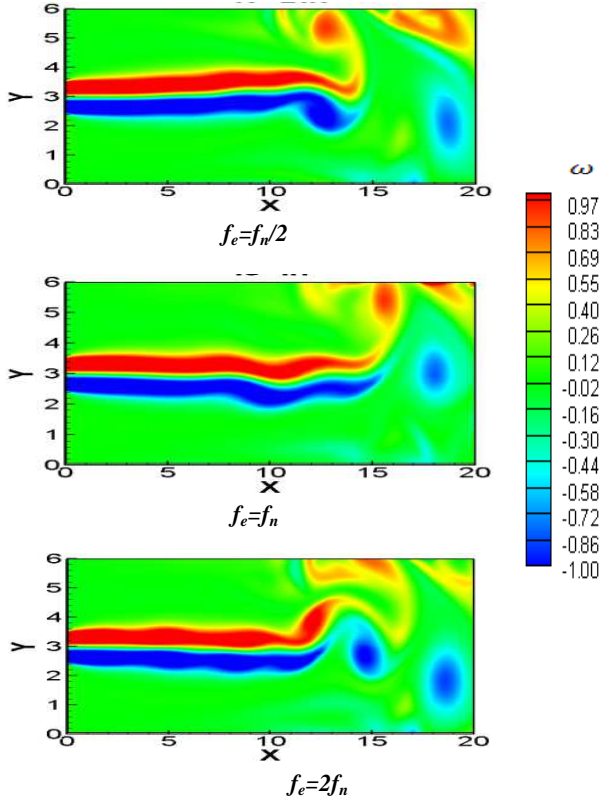


Fig. 10. Vorticity field ω for different excitation frequencies ($Re = 500$).

7. Effect of viscosity on the vortex structures

The Fig. 11 presents the behaviour of vortices in the domain is described by the evolution of the vorticity field for different Reynolds numbers. The excitation frequency is maintained equal to the natural frequency of instability f_n . The concentration of iso-contours ψ near the jet axis is due to the relatively high intensity of the vortex initially formed. The formation and dissociation of vortex persist near the downstream for a small viscosity. Further downstream, the layer is rolled to form a vortex structure that grows progressively as it is advected by the mean motion of the fluid. By increasing the Reynolds number, the flow sensitivity to any external perturbation becomes

bigger. The viscosity has a role to stabilize the flow. The width of the shear layer formed in the flow increases continuously with the viscosity and produces large scale vortices while keeping the same structure. These varicose structures in the upstream propagate to the downstream and they give a sinuous mode.

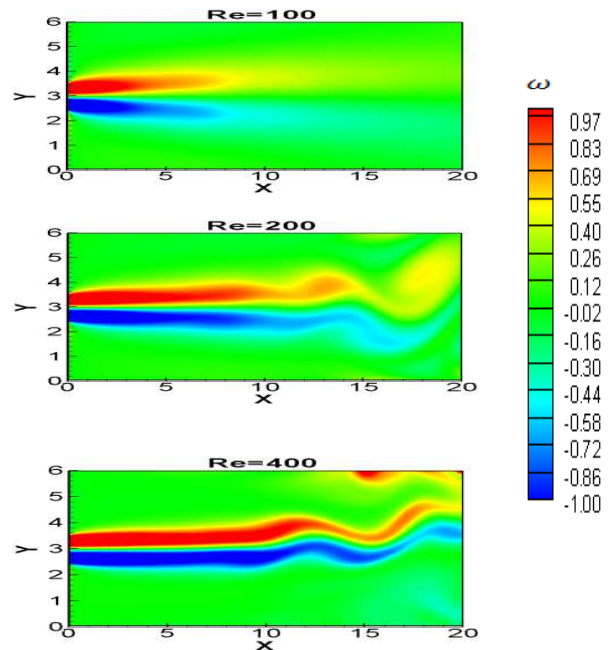


Fig. 11. Vorticity field ω for different Reynolds numbers

8. Conclusions

In order to observe a non-linear effect on vortex windings, a control study of the jet has been carried. It consists to imposing a sinusoidal perturbation at the inlet. The excitation frequency was chosen equal to a harmonic or sub-harmonic of the natural frequency. We found, for a perturbations realized at different frequencies, the response frequency increases with increasing excitation frequency. The forcing at the natural frequency amplifies the harmonics or sub harmonic of the excitation frequency. An attenuation of the amplitude is observed for large Reynolds numbers. The presence of harmonic and subharmonics frequencies corresponding to the maximum amplitude

of the wave instability made it possible to highlight the nonlinear phenomenon.

We found an agreement between our results and those existing in the literature in some intervals of disturbance frequency. The results of our simulations are in agreement with those from the work of some authors. Nevertheless, determining the boundaries of the frequencies intervals remains an issue to be examined.

Acknowledgments

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