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Finite element model for sound transmission analysis through a double panel inserted in an infinite baffle

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Abstract: This paper presents a finite element model for sound transmission analysis through a double panel inserted in an infinite baffle. The proposed model is derived from a multi-field variational principle involving structural displacement and acoustic pressure inside the fluid cavity. To solve the vibro-acoustic problem, the plate displacements are expanded as a modal summation of the plate's eigenfunctions in vacuo. Similarly, the cavity pressure is expanded as a summation of the modes of the cavity with rigid boundaries. Then, an appropriate reduced-order model is introduced. The structure is excited by a plane wave at the source side. The radiated sound power is calculated by means of a discrete solution of the Rayleigh Integral. Fluid loading is neglected. An example of the normal sound transmission loss of a double aluminum panel is shown. This example illustrates the accuracy and the versatility of the proposed reduced order model, especially in terms of prediction of sound transmission.

Key words: Fluid-structure, finite element, modal reduction, double panel, sound transmission.

1. Introduction

Double-wall structures are widely used in noise control due to their superiority over single-leaf structures in providing better acoustic insulation. Typical examples include double glazed windows, fuselage of airplanes, vehicles, etc. Different theoretical, experimental and numerical approaches have been investigated to predict the sound transmission through double walls. In [1, 2, 3] theoretical approaches are proposed for the derivation of the transmission sound factor of double panels of infinite size exposed to a random sound field as a function of frequency and angle of incidence. For a finite panels size, a theoretical study, based on Fourier series expansions, on the vibroacoustic performance of a rectangular double-panel partition clamp mounted in an infinite acoustic rigid baffle is presented in [4]. Experimental evaluation of sound transmission through single, double and triple glazing can be found in [5, 6, 7, 8]. Regarding the numerical prediction approaches, several methods are available in the literature, such as the finite element method (FEM), the boundary element method (BEM), the Statistical Energy Analysis (SEA), etc. In [9] FEM is applied to study the viscothermal fluid effects on vibro-acoustic behaviour of double elastic panels. The FEM is applied in [10] by the authors for the different layers of the sound barrier coupled to a variational BEM to account for fluid loading. In [11], the SEA is used for predicting sound transmission through double walls and for computing the non-resonant loss factor. For all these approaches, the choice of the numerical method is related to the computational cost and the frequency band to be treated.

This paper describes a finite element model for the sound transmission analysis through a double panel inserted in an infinite baffle. This model is derived from a variational principle involving structural displacement and acoustic pressure in the fluid cavity. To solve the vibro-acoustic problem, the direct solution can be considered only for small model sizes. This has severe limitations in attaining adequate accuracy and wider frequency ranges of interest. A reduced order-model is then proposed to solve the problem at a lower cost. The proposed methodology, based on a normal mode expansion, requires the computation of the uncoupled structural and acoustic modes. The uncoupled structural modes are the modes of the panels without fluid pressure loading at fluid-structure interface, whereas the uncoupled acoustic modes are the cavity modes with rigid wall boundary conditions at the fluid-structure interface. The effects of the higher modes of each subsystem can be taken into account through an appropriate so-called "static correction", however this method is out of the scope of this work.

As a next step, the sound transmission through double walls with air cavity is investigated. When the normal velocity distribution of the panel is known, the acoustic pressure field generated in the outward direction of the two plates can be calculated with the so-called Rayleigh integral for two-dimensional sound radiation. For this purpose, it is assumed that the double wall panel is placed in an infinite baffle. The normal incidence sound transmission is chosen in order to evaluate the acoustic performances and the sound insulation of the double wall. Example of the normal sound transmission loss of a double aluminum panel is shown in order to illustrate the accuracy and the versatility of the proposed reduced order model.

2. Finite element formulation of the coupled problem

2.1 Local equations

Consider a double-wall structure shown in Fig. 1. Each wall occupies a domain Ω_{Si}, $i \in \{1, 2\}$ such that $\Omega_S = (\Omega_{S1}, \Omega_{S2})$ is a partition of the whole structure domain. A prescribed force density $\mathbf{F}^d$ is applied to the external boundary Γ_t of Ω_S and a prescribed displacement $\mathbf{u}^d$ is applied on a part Γ_u of Ω_S. The two structures are separated by an acoustic enclosure filled with a compressible and inviscid fluid occupying the domain Ω_F. The cavity walls are rigid except those in contact with the flexible wall structures noted Σ.

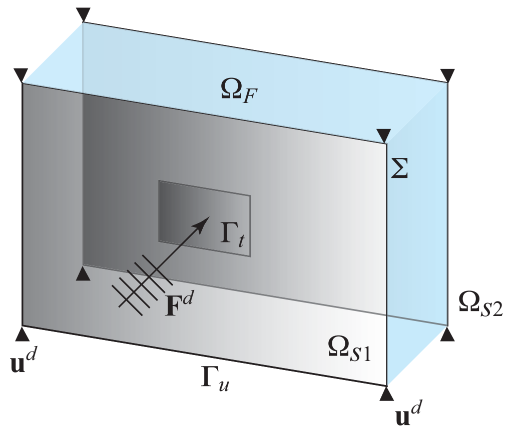

Fig. 1 Double wall structure.

The harmonic local equations of this structural-acoustic coupled problem can be written in terms of structure displacement $\mathbf{u}$ and fluid pressure field p [12, 13]

$$\operatorname{div} \sigma(\mathbf{u}) + \rho_S \omega^2 \mathbf{u} = \mathbf{0} \text{ in } \Omega_S \tag{1}$$

$$\sigma(\mathbf{u})\mathbf{n}_S = \mathbf{F}^d \qquad \text{on } \Gamma_t \tag{2}$$

$$\mathbf{u} = \mathbf{u}^d \qquad \text{on } \Gamma_u \tag{3}$$

$$\sigma(\mathbf{u})\mathbf{n}_S = p\mathbf{n} \qquad \text{on } \Gamma_u \qquad (4)$$

$$\Delta p + \frac{\omega^2}{c_F^2} p = 0 \qquad \text{in } \Omega_F \qquad (5)$$

$$\nabla p \cdot \mathbf{n} = \rho_F \omega^2 \mathbf{u} \cdot \mathbf{n} \qquad \text{on } \Sigma \qquad (6)$$

where ω is the angular frequency, $\mathbf{n}_S$ and $\mathbf{n}$ are the external unit normal to Ω_S and Ω_F; ρ_S and ρ_F are the structure and fluid mass densities; c_F is the speed of sound in the fluid; and σ is the structure stress tensor.

Equation (1) corresponds to the elastodynamic equation in the absence of body force; (2) and (3) are the prescribed mechanical boundary conditions; (4) results from the action of pressure forces exerted by the fluid on the structure; (5) is the Helmholtz equation; and (6) is the contact condition for the fluid on Σ.

2.2 Variational formulation

The variational formulation of the problem is obtained using the test-function method. For this purpose, we introduce the spaces C_u and C_p of sufficiently smooth functions associated with the field variables $\mathbf{u}$ and p respectively.

Let $\delta\mathbf{u}$ be the test function associated to $\mathbf{u}$, belonging to the admissible space $C_u^* = \{\delta\mathbf{u} \in C_u \,/\, \delta\mathbf{u} = \mathbf{0} \text{ on } \Gamma_u\}$. Multiplying (1) by $\delta\mathbf{u} \in C_u^*$, applying Green's formula, and finally taking (2) and (4) into account, we have:

$$\int_{\Omega_S} \sigma(\mathbf{u}) : \varepsilon(\delta\mathbf{u}) \mathrm{d}v - \int_{\Sigma} p\mathbf{n} \cdot \delta\mathbf{u} \mathrm{d}s - \omega^2 \int_{\Omega_S} \rho_S \mathbf{u} : \delta\mathbf{u} \mathrm{d}v = \int_{\Gamma_t} \mathbf{F}^d \cdot \delta\mathbf{u} \mathrm{d}v \quad \forall \delta\mathbf{u} \in C_u^* \qquad (7)$$

Similarly, let δp be the test function, associated to p, belonging to the admissible space C_p. Multiplying (5) by $\delta p \in C_p$, applying Green's formula, and finally taking (6) into account, we obtain:

$$\frac{1}{\rho_F} \int_{\Omega_F} \nabla p \cdot \nabla \delta p \mathrm{d}v - \omega^2 \int_{\Sigma} \mathbf{u} \cdot \mathbf{n} \delta p \mathrm{d}s - \frac{\omega^2}{\rho_F c_F^2} \int_{\Omega_F} p \delta p \mathrm{d}v = 0 \quad \forall \delta p \in C_0 \qquad (8)$$

Thus, the variational unsymmetric formulation of the fluid/elastic structure coupled problem consists in finding $\mathbf{u} \in C_u$ such that $\mathbf{u} = \mathbf{u}^{\mathbf{d}}$ on Γ_u and $p \in C_p$, satisfying (7) and (8), with appropriate initial conditions. The symmetrization of this formulation can be obtained through the introduction of an intermediate unknown field, namely the fluid displacement potential field [14, 15].

After discretizing by the finite element method the bilinear forms in (7) and (8), we obtain the following matrix system of the coupled problem:

$$\left[\begin{pmatrix} \mathbf{K}_u & -\mathbf{C}_{up} \\ \mathbf{0} & \mathbf{K}_p \end{pmatrix} - \omega^2 \begin{pmatrix} \mathbf{M}_u & \mathbf{0} \\ \mathbf{C}_{up}^T & \mathbf{M}_p \end{pmatrix}\right] \begin{pmatrix} \mathbf{U} \\ \mathbf{P} \end{pmatrix} = \begin{pmatrix} \mathbf{F} \\ \mathbf{0} \end{pmatrix} \qquad (9)$$

where $\mathbf{U}$ and $\mathbf{P}$ are the vectors of nodal values of $\mathbf{u}$ and p respectively. The submatrices of (9) are given by:

$$\int_{\Omega_S} \sigma(\mathbf{u}) : \varepsilon(\delta\mathbf{u})\mathrm{d}v \Rightarrow \delta\mathbf{U}^T\mathbf{K}_u\delta\mathbf{U}$$
$$\int_{\Omega_S} \rho_S \mathbf{u} : \delta\mathbf{u}\mathrm{d}v \Rightarrow \delta\mathbf{U}^T\mathbf{M}_u\delta\mathbf{U}$$
$$\int_{\Gamma_t} \mathbf{F}^d \cdot \delta\mathbf{u}\mathrm{d}v \Rightarrow \delta\mathbf{U}^T\mathbf{F}$$
$$\int_{\Sigma} p\mathbf{n} \cdot \delta\mathbf{u}\mathrm{d}s \Rightarrow \delta\mathbf{U}^T\mathbf{C}_{up}\delta\mathbf{P}$$
$$\int_{\Sigma} \mathbf{u} \cdot \mathbf{n}\delta p\mathrm{d}s \Rightarrow \delta\mathbf{P}^T\mathbf{C}_{up}^T\delta\mathbf{U}$$
$$\frac{1}{\rho_F}\int_{\Omega_F} \nabla p \cdot \nabla\delta p\mathrm{d}v \Rightarrow \delta\mathbf{P}^T\mathbf{K}_p\delta\mathbf{P}$$
$$\frac{1}{\rho_F c_F^2}\int_{\Omega_F} p\delta p\mathrm{d}v \Rightarrow \delta\mathbf{P}^T\mathbf{M}_p\delta\mathbf{P}$$

Remark:

The static response of the fluid to a prescribed wall normal displacement $\mathbf{u}\cdot\mathbf{n}$ on the fluid-structure interface Σ is obtained from the following constraint

$$\rho_F c_F^2 \int_{\Sigma} \mathbf{u}\cdot\mathbf{n}\mathrm{d}s + \int_{\Omega_F} p\mathrm{d}v = 0 \qquad (10)$$

This constraint has to be added to the variational formulation of the problem ((7) and (8)) in order to regularize the zero frequency situation (see [14] for more details).

From (10), the constant static pressure is given by

$$p^s = -\frac{\rho_F c_F^2}{|\Omega_F|}\int_{\Sigma} \mathbf{u}\cdot\mathbf{n}\mathrm{d}s \qquad (11)$$

in which $|\Omega_F|$ denotes the volume occupied by the domain Ω_F.

3. Reduced order model

In this section, we introduce a reduced-order formulation of the variational equations (7) and (8) by a Ritz-Galerkin projection on two bases spanning the admissible spaces C_u and C_p. For C_u, we use the *in vacuo* structural modes. Concerning C_p, the basis is formed by the eigenmodes of the Helmholtz equation with rigid boundary condition. In the sequel, instead of starting from the variational formulation, we will carry the projection directly on the discretized system (9).

3.1 Eigenmodes of the structure in vacuo

In a first phase, the first N_s eigenmodes of the structure in vacuo are obtained from

$$[\mathbf{K}_u - \omega_{si}^2\mathbf{M}_u]\mathbf{Y}_{si} = \mathbf{0} \text{ for } i \in \{1,\cdots,N_s\}$$

where $(\omega_{si}, \mathbf{Y}_{si})$ are the natural frequency and eigenvector for the *i*-th structural mode. These modes verify the following orthogonality properties

$$\mathbf{Y}_{si}^T\mathbf{M}_u\mathbf{Y}_{sj} = \delta_{ij} \text{ and } \mathbf{Y}_{si}^T\mathbf{K}_u\mathbf{Y}_{sj} = \omega_{si}^2\delta_{ij}$$

where δ_{ij} is the Kronecker symbol and $\mathbf{Y}_{sj}$ have been normalized with respect to the structure mass matrix.

3.2 Eigenmodes of the internal acoustic cavity with rigid walls

In this second phase, the first N_f eigenmodes of the acoustic cavity with rigid boundary conditions are obtained from the following equation

$$[\mathbf{K}_p - \omega_{fi}^2\mathbf{M}_p]\mathbf{Y}_{fi} = \mathbf{0} \text{ for } i \in \{1,\cdots,N_f\}$$

where $(\omega_{fi}, \mathbf{Y}_{fi})$ are the natural frequency and eigenvector for the *i*-th acoustic mode. These modes verify the following orthogonality properties

$$\mathbf{Y}_{fi}^T\mathbf{M}_p\mathbf{Y}_{fj} = \delta_{ij} \text{ and } \mathbf{Y}_{fi}^T\mathbf{K}_p\mathbf{Y}_{fj} = \omega_{fi}^2\delta_{ij}$$

where $\mathbf{Y}_{fj}$ have been normalized with respect to the fluid mass matrix.

3.3 Modal expansion of the general problem

By introducing the matrices $\mathbf{Y}_s = [\mathbf{Y}_{s1} \ \cdots \ \mathbf{Y}_{sN_s}]$ of size (M_s, N_s) and $\mathbf{Y}_f = [\mathbf{Y}_{f1} \ \cdots \ \mathbf{Y}_{fN_f}]$ of size (M_f, N_f) corresponding to the uncoupled bases (M_s and M_f are the total number of degrees of freedom in the finite elements model associated to the structure and the acoustic domains respectively), the displacement and pressure are sought as

$$\mathbf{U} = \mathbf{Y}_s \mathbf{q}_s(t) \text{ and } \mathbf{P} = \mathbf{Y}_f \mathbf{q}_f(t) \quad (12)$$

where the vectors $\mathbf{q}_s = [q_{s1} \ \cdots \ q_{sN_s}]^T$ and $\mathbf{q}_f = [q_{f1} \ \cdots \ q_{fNf}]^T$ are the modal amplitudes of the structure displacement and the fluid pressure respectively.

Substituting these relations into (9) and pre-multiplying the first row by $\mathbf{Y}_s^T$ and the second one by $\mathbf{Y}_f^T$, we obtain the equation

$$\begin{bmatrix} \mathbf{Y}_s^T \mathbf{K}_u \mathbf{Y}_s & -\mathbf{Y}_s^T \mathbf{C}_{up} \mathbf{Y}_f \\ \mathbf{0} & \mathbf{Y}_f^T \mathbf{K}_p \mathbf{Y}_f \end{bmatrix} \begin{bmatrix} \mathbf{q}_s \\ \mathbf{q}_f \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{Y}_s^T \mathbf{M}_u \mathbf{Y}_s & \mathbf{0} \\ \mathbf{Y}_f^T \mathbf{C}_{up}^T \mathbf{Y}_s & \mathbf{Y}_f^T \mathbf{M}_p \mathbf{Y}_f \end{bmatrix} \begin{bmatrix} \mathbf{q}_s \\ \mathbf{q}_f \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_s^T \mathbf{F} \\ \mathbf{0} \end{bmatrix} \quad (13)$$

This matrix equation represents the reduced order model of the structural acoustic. If only few modes are kept for the projection, the size of this reduced order model ($N_s \times N_f$) is much more smaller than the initial one ($M_s \times M_f$). Equation (13) can be also written in the following form of coupled differential equations:

- N_s mechanical equations

$$-\omega^2 q_{si} + 2i\omega\omega_{si}\xi_{si} q_{si} + \omega_{si}^2 q_{si} - \sum_{j=1}^{N_f} \beta_{ij} q_{fj} = F_i$$

- N_f acoustic equations

$$-\omega^2 q_{fi} + 2i\omega\omega_{fi}\xi_{fi} q_{fi} + \omega_{fi}^2 q_{fi} - \omega^2 \sum_{j=1}^{N_s} \beta_{ij} q_{sj} = 0$$

where $F_i = \mathbf{Y}_{si}^T \mathbf{F}$ is the mechanical excitation of the i-th mode; $\beta_{ij} = \mathbf{Y}_{si}^T \mathbf{C}_{up} \mathbf{Y}_{fj}$ is the fluid-structure modal coupling coefficient; ξ_{si} and ξ_{fi} are the introduced modal damping coefficients for structure and fluid respectively.

4. Acoustic indicators

In order to evaluate the acoustic performances and the sound insulation property of the double-wall panels, the radiated sound power (π_t) and the normal incidence sound transmission (nSTL) are used as acoustic indicators in this work.

4.1 Radiated sound power

The radiated (or transmitted) sound power through the area S_2 of the panel Ω_{S2} is given by:

$$\pi_t(\omega) = \frac{1}{2} \mathrm{Re}\left(\int_{S_2} p(\omega, \mathrm{G}) v_n^*(\omega, \mathrm{G}) \mathrm{d}S \right) \quad (14)$$

where G is a point on the plate surface S_2, p is the sound pressure applied as an external loading, v_n is the normal velocity ($*$ denotes the complex conjugate) and Re is the real part of the expression.

For a flat plate embedded in an infinite rigid plane baffle and radiating in a semi infinite fluid, p can be obtained using the Rayleigh Integral [3]:

$$p(\omega, \mathrm{M}) = \rho_0 \frac{i\omega}{2\pi} \int_{S_2} v_n(\omega, \mathrm{G}) \frac{e^{-ikr}}{r} \mathrm{d}S$$

where ρ_0 is the mass density of the external acoustic domain, k is the wave number expressed as ω/c_0, c_0 is the acoustic speed of sound, M is a point inside the external acoustic domain and $v_n(\omega,\mathrm{G})$ is the normal velocity at point G expressed as $v_n(\omega,\mathrm{G}) = \mathbf{v}(\omega,\mathrm{G}) \cdot \mathbf{n}_S$. Note that the normal velocity distribution on the structure can be easily obtained from the previous finite element formulation.

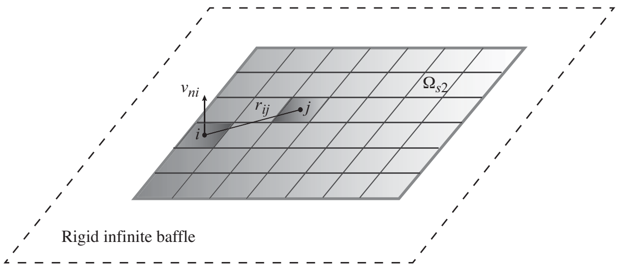

Fig. 2 Subdivision of the panel Ω_{S2} into elemental radiators.

As shown in Fig. 2 the baffled panel is divided into a grid of R rectangular elements with equal size whose transverse vibrations are specified in terms of the normal velocities at their centre positions. Assuming that the dimensions of the element are small compared with both the structural wavelength and the acoustic wavelength, the total radiated sound power (equation (14)) can then expressed as the summation of the powers radiated by each element, so that

$$\pi_t = \frac{S_e}{2}\mathrm{Re}(\mathbf{v}_n^H \mathbf{p}) \tag{15}$$

where the superscript H denotes the hermitian transpose, $\mathbf{v}_n$ and $\mathbf{p}$ are the vectors of complex amplitudes of the normal volume velocity and acoustic pressure in all elements and S_e is the area of each element. The pressure on each element is generated by the vibrations of all elements of the panel. The vector of sound pressure can therefore be expressed by the impedance matrix relation

$$\mathbf{p} = \mathbf{Z}\mathbf{v}_n \tag{16}$$

where $\mathbf{Z}$ is the matrix incorporating the point and transfer acoustic impedance terms over the grid of elements into which the panel has been subdivided: $Z_{ij} = (i\omega\rho_0 S_e / 2\pi r_{ij})e^{-ikr_{ij}}$ (r_{ij} is the distance between the centers of the *i*-th and *j*-th elements). Note that, because of reciprocity, the impedance matrix $\mathbf{Z}$ is symmetric. Substituting (16) into the expression for the total radiated sound power given in (15), we obtain

$$\pi_t = \frac{S_e}{2}\mathrm{Re}(\mathbf{v}_n^H \mathbf{Z}\mathbf{v}_n) = \frac{S_e}{4}\mathrm{Re}(\mathbf{v}_n^H[\mathbf{Z} + \mathbf{Z}^H]\mathbf{v}_n) = \mathbf{v}_n^H \mathbf{R}\mathbf{v}_n$$

The matrix $\mathbf{R}$ is defined as the "radiation resistance matrix" for the elementary radiators which, for the baffled panel, is given by

$$\mathbf{R} = \frac{\omega^2\rho_0 S_e^2}{4\pi c_0}\begin{bmatrix} 1 & \frac{\sin(kr_{12})}{kr_{12}} & \cdots & \frac{\sin(kr_{1R})}{kr_{1R}} \\ \frac{\sin(kr_{21})}{kr_{21}} & 1 & \cdots & \frac{\sin(kr_{2R})}{kr_{2R}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\sin(kr_{R1})}{kr_{R1}} & \frac{\sin(kr_{R2})}{kr_{R2}} & \cdots & 1 \end{bmatrix}$$

This method can be applied to any plane surface in an infinite baffle, independently of the boundary conditions. It only requires the knowledge of the surface geometry, the characteristics of the fluid and the velocity field distribution. In this work, a finite element approach is used to evaluate this velocity field by using a sufficient number of discrete radiating elements according to the smallest wavelength to be observed.

4.2 Normal incidence sound transmission

The normal incidence sound transmission of the double-wall sandwich panels is investigated in this section using Rayleigh Integral method described below. It is evaluated using the following formula:

$$nSTL = 10\, log \frac{\pi_i}{\pi_t}$$

where π_i and π_t are the incident and transmitted acoustic power respectively. For normal plane wave applied to plate Ω_{S1}, the incident sound power is given by:

$$\pi_i = \frac{\left|P_{inc}\right|^2 S_1}{2\rho_0 c_0}$$

where P_{inc} represents the normal incident sound pressure amplitude and S_1 is the area of the whole panel Ω_{S1}.

5. Numerical example

In this last section, numerical results, obtained with a Matlab program developed by the authors, are proposed in order to validate and analyze results computed from the proposed formulations. The example concerns sound transmission through a double-plate system filled with air. In this example, we analyze the air gap effect on the natural vibration of the coupled system and the sound attenuation. The accuracy of model predictions is checked against existing test data.

The problem under consideration is shown in Fig. 3. A normal incidence plane wave excites a double-plate system filled with air (density ρ_F = 1.21 kg/m^3 and speed of sound c_F = 340 m/s). The plane wave has a pressure amplitude of 1 N/m^2 and is applied to plate 1 as the only external force to the system. The plates are identical and simply supported with thicknesses of 1 mm. The density of the plates is 2814 kg/m^3, the Young's modulus is 71 GPa, the Loss factor is 0.01 and Poisson ratio 0.33. The surrounding fluid is the air. This example was originally proposed by Panneton in [16].

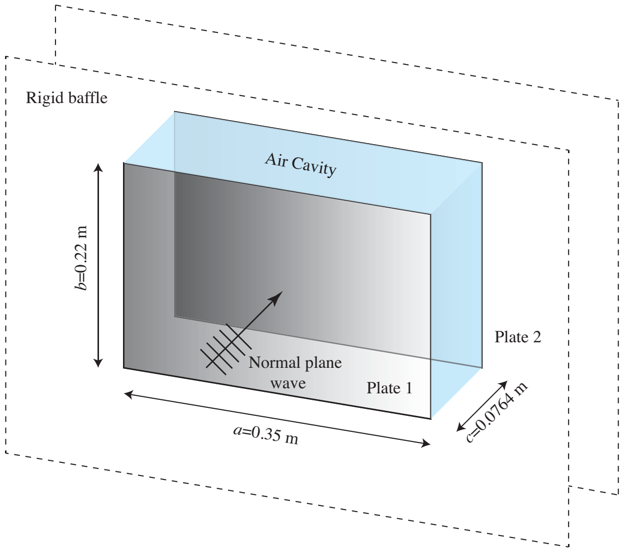

Fig. 3 Double-plate system filled with air: geometric data.

Concerning the finite element discretization, we have used, for the structural part, 10×10 rectangular plate elements for each panel. The acoustic cavity is discretized using $10 \times 10 \times 5$ hexahedric elements. The structural and acoustic meshes are compatible at the interface. For more details about these elements and the fluid-structure coupling element, we refer the reader to [13].

When the excitation is applied to the first plate, the second one vibrates and radiates sound caused by the coupling of air and plate 1. The normal incidence sound transmission loss is then computed using the Rayleigh's integral method which needs the finite element solution of surface velocities of plate 2.

For this purpose, the resolution of the coupled system is done with a modal reduction approach. In order to evaluate the number of structural and acoustic modes to keep in the modal projection, various simulations have been performed and compared to results of direct method. We consider that N_s = 20 and N_f = 20, which corresponds approximatively to the double of the frequency of interest ([0, 1000] Hz), is satisfying. In this respect, it should be noted that the resulting reduction of the computational effort using the reduced order method is very significant compared to those of the direct approach (about one minute for the reduced model and 34 minutes for direct approach).

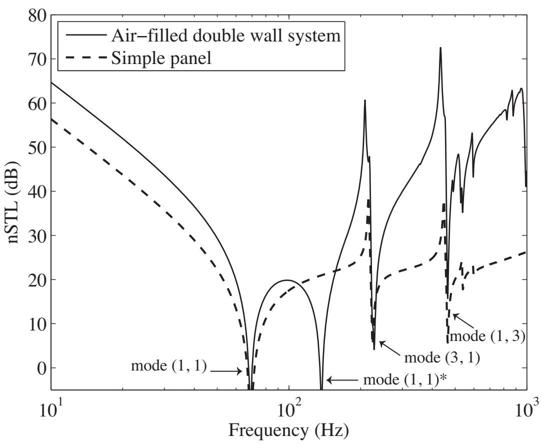

Fig. 4 Comparison of the normal incidence sound transmission (nSTL) through an air-filled double panel and a simple panel.

Fig. 4 shows the normal incidence transmission loss through a simply supported plate (dashed line). Due to the modal behavior of the plate, dips in the transmission loss curve are observed at its resonance frequencies (modes (1, 1), (3, 1) and (1, 3)). When a second plate is used to form an airtight cavity (continuous line), an increase in the transmission loss is achieved except in the region of the so-called plate-cavity-plate resonance (mode (1, 1)*). At this frequency, the two plates move out of phase with each other and the effect of the cavity on the plates is mostly one of added stiffness. This frequency is similar to the mass-air-mass resonance of unbounded double panels analyzed in the next paragraph. The frequencies and the mode shapes of these coupled modes are presented in Fig. 5. Note that the influence of several key parameters on the sound isolation capability of the double-panel configuration including panel dimensions, thickness of air cavity, elevation angle, and azimuth angle of incidence sound is not the purpose of this study and can be found in [4].

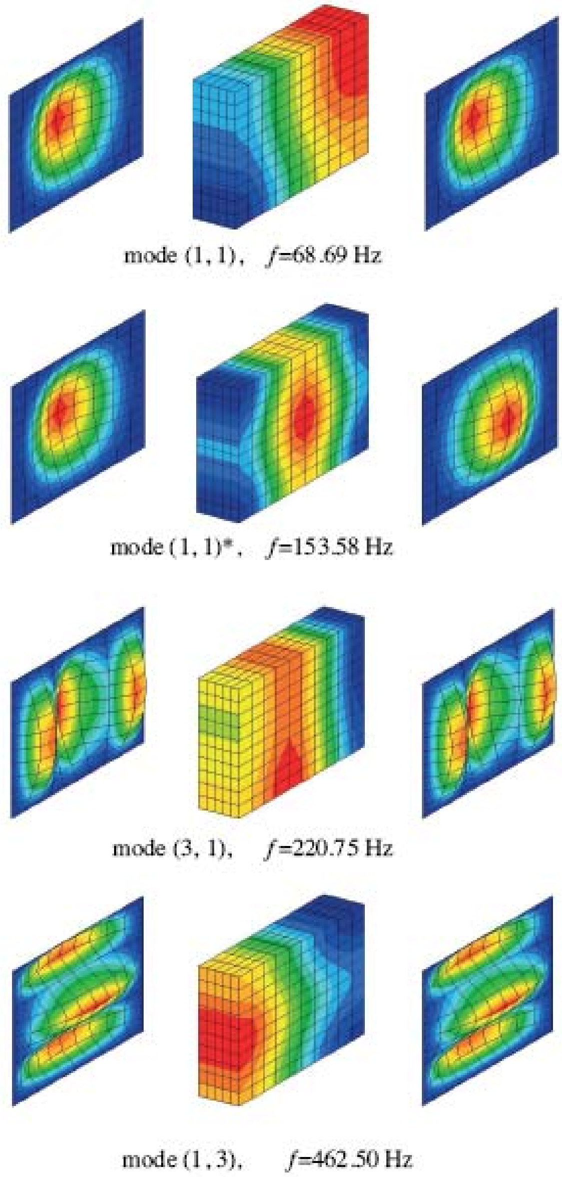

Fig. 5 Frequencies and the mode shapes of the first fourth coupled modes.

Fig. 6 presents a comparison of the normal incidence sound transmission through an air-filled finite double panel and an air-filled infinite double panel computed from an analytical solution given in [3, 17]. For unbounded panels, the first dip occurs at the mass-air-mass frequency (f_{mam}=181.63 Hz) given by the formula:

$$f_{mam} = \frac{1}{2\pi}\sqrt{\frac{\rho_F c_F^2}{d}\frac{m_{s1}+m_{s2}}{m_{s1}m_{s2}}}$$

where d is the panel spacing and m_{S1} and m_{S2} are the surface mass densities of the panels.

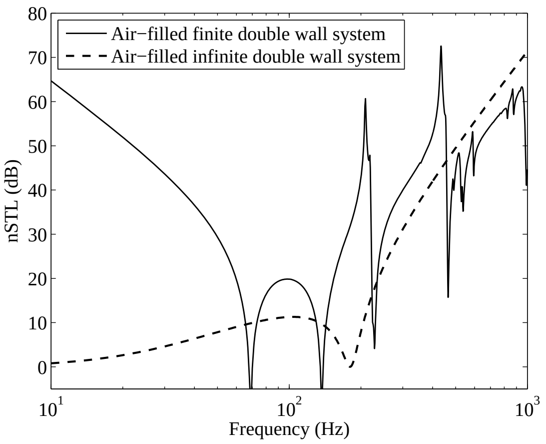

Fig. 6 Comparison of the normal incidence sound transmission (nSTL) through an air-filled finite double panel (finite element result) and an air-filled infinite double panel (analytical solution).

At this frequency, the two plates vibrate, as it were, on the stiffness of the air layer. For low frequencies up to the mass-air-mass frequency, the transmission loss follows the so-called mass law: the two plates are coupled in such a way that the plates vibrate as if they were a single plate with the total mass of the two plates and the transmission loss increasing with frequency at 6 dB per octave and 6 dB when the mass is doubled. It's clear from this comparison that the unbounded model is attractive to use for the prediction of global trends at higher frequencies, but is unsuitable to use for predictions in the small frequency bands and around eigenfrequencies of the double wall panel [17].

6. Conclusions

In this paper, a finite element formulation for sound transmission through double wall panels is presented. A reduced-order model, based on a normal mode expansion, is then developed. The proposed methodology requires the computation of the eigenmodes of the structure *in vacuo*, and the rigid acoustic cavity. Despite its reduced size, this model is proved to be very efficient for simulations of steady-state analyses of structural-acoustic coupled systems. The Rayleigh integral method is then used in order to estimate the sound transmission loss factor of the system. Examples are presented in order to validate and illustrate the efficiency of the proposed finite element formulation.

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