

Entropy Generation in Mixed Convection through a Horizontal Porous Channel

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Abstract: The numerical analysis of two-dimensional laminar mixed convection flow through a channel filled with saturated porous media under thermal gradient is investigated. The Darcy-Brinkman model is employed. The conservation equations are solved using a Control Volume Finite Element Method. Total entropy generation is investigated at global and local levels by varying the porosity from 1 to 0.2 at fixed values of $Ra=10^4$, $Re=10$, $Br^*=10^{-3}$. Results show the existence of dissipative structures.

Key words: Numerical method, Entropy generation, porous media, mixed convection.

Nomenclature

Da : Darcy number (K/H^2)

K : Permeability of the porous media (m^2)

g : gravitational acceleration ($m.s^{-2}$)

Ra: Rayleigh number in porous media
($\beta g \Delta T H^3 / \nu \cdot \alpha_{eff}$)

Re: Reynolds number (Hu_0/ν)

Pe: Peclet number ($Re.Pr$)

Br: Brinkman number ($Ec.Pr$)

Br*: modified Darcy-Brinkman number
(Br/Ω)

Ec: Eckert number ($u_0^2/c_p\Delta T$)

u_0 : average velocity ($m.s^{-1}$)

H: channel width (m)

L: length of the channel (m)

p: pressure nondimensionalized ($N.m^{-2}$)

P: dimensionless pressure

Pr: Prandtl number (μ_p/k_m)

t: time (s)

T: temperature (K)

T_0 : mean Temperature [$(T_h+T_c)/2$] (K)

ΔT : temperature difference (T_h-T_c)

$\langle Nu \rangle$: the space-averaged Nusselt number

S: dimensionless entropy generation

$\langle S \rangle$: time average entropy generation

\mathbf{v} : dimensional velocity vector ($m.s^{-1}$)

\mathbf{V} : dimensionless velocity vector

v_x, v_y : velocity components in x and y directions respectively ($m.s^{-1}$)

V_x, V_y : Dimensionless velocity components in X and Y directions respectively

x,y: Cartesian coordinates (m)

X,Y : dimensionless Cartesian coordinates

Greek symbols

β : Thermal expansion coefficient (K^{-1})

ε : porosity of the porous medium

θ : dimensionless temperature

Θ : dimensionless period

ρ : mass density ($kg.m^{-3}$)

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σ : specific heat capacities ratio $((\rho c)_m/(\rho c)_f)$ Λ : viscosity ratio (μ_{eff}/μ) μ : dynamic viscosity $(\text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-1})$ ν : kinematic viscosity $(\text{m}^2\cdot\text{s}^{-1})$ τ : dimensionless time Ω : dimensionless temperature difference $(\Delta T/T_0)$ **Subscripts**

a: dimensionless

c: cold wall

F: fluid friction

H: heat transfer

h: hot wall

l: local

t: total

m: porous media

f: fluid

s: solid

1. Introduction

Mixed convection with steady laminar flow in a parallel plates channel is a classical problem, and it is the subject of hundreds of papers. The studies are analyzed either by numerical methods or analytical approaches or by experimental procedures. We report here some works dealing with forced convection and free convection in the presence of porous medium. Nield and Bejan [1] and Bejan et al.[2] have made analysis of the forced convection in the channels parallel plate with a saturated porous medium. Al-Hadhrami et al. [3] investigated the combined free and forced convection of a fully developed Newtonian fluid within a vertical channel composed of porous media when viscous dissipation effects are taken into consideration. The second law of thermodynamics is applied to investigate the irreversibilities in terms of entropy generation. Abu-Hijleh [4] presented a numerical analysis of entropy generation in a porous cylinder due to heat transfer. Baytas [5] analyzed the entropy production for natural and forced convection in a porous medium. Tasnim et al.[6] presented an analytical work to study the first and second laws (of thermodynamics) characteristics of flow and heat transfer inside a vertical channel made of two parallel plates embedded in a porous medium and under the action of transverse magnetic field. Mahmud and Fraser [7] gave a detailed analysis of entropy generation due to the mixed convection and the radiation in a vertical porous channel. Later, Mahmud and Fraser [8] have numerically and analytically

investigated entropy production in a porous channel under thermal and viscous effect. They studied the variation of velocity and temperature profiles, Nusselt number, rate of entropy generation and Bejan numbers depending on the number of Darcy. They showed that the Darcy number is an important parameter because it provides a relative measure of the permeability of porous media. Their results show that the increase in the Darcy number reduces the flatness of the velocity profile at the centerline of the channel and flat velocity distribution for low Darcy number. In more recent paper, entropy generation due to forced convection in a porous medium was analytically investigated by Hooman et al. [9] and numerically by Hooman and Ejlali [10] and Hooman et al. [11]. Guo et al. [12] numerically studied the effect of viscous dissipation on entropy generation for laminar flow region for different fluids in curved square microchannels. Furthermore, entropy generation in a vertical square channel packed with saturated porous media, and subjected to differentially heat isothermal walls was numerically investigated by Abdulhassan et al. [13]. He showed that the value of the entropy generation number decreases as the Reynolds number, Darcy number increases and Eckert number decreases. The results indicate that irreversibility due to fluid friction dominate for higher Darcy numbers, while as Darcy decrease, the irreversibility dominates due to the heat transfer. Recently, Mchirgui et al. [14] numerically investigated entropy generation in double diffusive convection through a square porous cavity,

filled with a binary perfect gas mixture. It was found that entropy generation increases with the decrease of the Darcy number, for both cases of cooperative and opposite buoyancy forces.

2. Mathematical formulation

The steady laminar flow of a Newtonian fluid within a horizontal porous channel as depicted in Fig.1. At the channel inlet, the normal component of velocity is assumed to be zero, and a fully developed parabolic profile for the axial velocity is deployed. To avoid discontinuity, the temperature of incoming stream is assumed to vary linearly from T_h at the bottom wall to T_c at the upper wall. The channel is composed of a vertical thermal gradient.

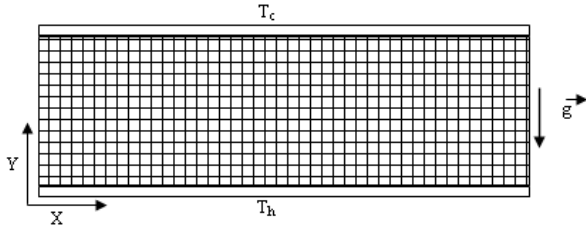


Fig.1 Schematic diagram of the problem.

The principle of this article is thermodynamics analysis of the incompressible viscous laminar flow < through a channel filled with porous media. Expressions for dimensionless equations for continuity, momentum, and energy are given by:

$$\text{div}(V) = 0 \quad (1)$$

$$\frac{\partial V_x}{\partial \tau} + \text{div}(J_{V_x}) = -\varepsilon \frac{\partial P}{\partial X} - \frac{\varepsilon}{Da \cdot Re} V_x \quad (2)$$

$$\frac{\partial V_y}{\partial \tau} + \text{div}(J_{V_y}) = -\varepsilon \frac{\partial P}{\partial Y} - \frac{\varepsilon}{Da \cdot Re} V_y + \frac{Ra \cdot \varepsilon}{Re \cdot Pe} \theta \quad (3)$$

$$\sigma \frac{\partial \theta}{\partial \tau} + \text{div}(J_\theta) = 0 \quad (4)$$

where : $J_{V_x} = \frac{1}{\varepsilon} V_x V - \frac{\Lambda \varepsilon}{Re} \text{grad}(V_x)$

$$J_{V_y} = \frac{1}{\varepsilon} V_y V - \frac{\Lambda \varepsilon}{Re} \text{grad}(V_y)$$

$$J_\theta = \theta V - \frac{1}{Re \cdot Pr} \text{grad}(\theta)$$

The governing equations are established using the following dimensionless variables:

$$\tau = \frac{t}{H} u_0 ; X = \frac{x}{H} ; Y = \frac{y}{H} ; V_x = \frac{v_x}{u_0} ; V_y = \frac{v_y}{u_0} ; P = \frac{p}{\rho_0 u_0^2} ; \theta = \frac{T - T_c}{T_h - T_c} \quad (5)$$

The boundary and initial conditions appropriate to laminar flow within the differential heated porous channel are:

$$0 \leq X \leq L/H ; Y = 0 ; V_x = V_y = 0 ;$$

$$\theta = 1 \quad 0 \leq X \leq L/H ; Y = 1 ; V_x = V_y = 0 ; \theta = 0$$

$$X = 0 ; 0 \leq Y \leq 1 ; V_x = 6Y(1-Y) ; V_y = 0 ; \theta = 1 - Y \quad (6)$$

$$X = L/H ; 0 \leq Y \leq 1 ; \frac{\partial \phi}{\partial \tau} + \frac{\partial \phi}{\partial X} = 0, \int_0^1 V_x dY = 1, (\phi = V_x, V_y)$$

$$\text{At } \tau = 0 ; V_x = V_y = 0 ; P = 0 ; \theta = 0.5 - X$$

3. Entropy generation

The expression of the volumetric entropy generation in mixed convection through a porous medium in 2D approximation is given by:

$$s = \frac{k_m}{T_0^2} \left[\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 \right] + \frac{\mu}{T_0 \cdot K} (v_x^2 + v_y^2) + \frac{\mu}{T_0} \left[2 \left(\frac{\partial v_x}{\partial x} \right)^2 + 2 \left(\frac{\partial v_y}{\partial y} \right)^2 + \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right)^2 \right] \quad (7)$$

According to Tasnim et al. [6] and Mahmud and Fraser [7], one can find the dimensionless form of local entropy generation in porous media as:

$$S_{l,a} = S_{l,a,H} + S_{l,a,D} + S_{l,a,F} \quad (8)$$

The first term on the right-hand side of (8) represents the heat transfer part of local entropy generation ($S_{l,a,H}$), the second part is the Darcy viscous entropy generation ($S_{l,a,D}$) and the third part is the

clear fluid viscous entropy generation ($S_{l,a,F}$). They are given by:

$$S_{l,a,H} = \left(\frac{\partial\theta}{\partial X}\right)^2 + \left(\frac{\partial\theta}{\partial Y}\right)^2 \quad (9)$$

$$S_{l,a,D} = \frac{Br^*}{Da} (V_x^2 + V_y^2) \quad (10)$$

$$S_{l,a,F} = Br^* \left[2\left(\frac{\partial V_x}{\partial X}\right)^2 + 2\left(\frac{\partial V_y}{\partial Y}\right)^2 + \left(\frac{\partial V_x}{\partial Y} + \frac{\partial V_y}{\partial X}\right)^2 \right] \quad (11)$$

Where Da and Br^* are the Darcy number and the modified Darcy–Brinkman number respectively. The dimensionless total entropy generation for the entire channel is obtained by integrating (8):

$$S_t = \int_0^1 \int_0^1 S_{l,a} dx dy \quad (12)$$

From the expression for total entropy generation number (12), the time-averaged total entropy generation can be evaluated using the following equation:

$$\langle S_t \rangle = \frac{1}{\Theta} \int_0^\Theta S_t d\tau \quad (13)$$

The thermal heat flux exchanged between the walls and the flow is characterized by the space-averaged Nusselt number evaluated as follows:

$$\langle Nu \rangle = \frac{1}{L/H} \int_0^{L/H} Nu dx \quad (14)$$

where Nu is the local Nusselt number defined as:

$$Nu = \left| \frac{\partial\theta}{\partial Y} \right| \quad (15)$$

4. Numerical procedure

A modified version of Control Volume Finite-Element Method (CVFEM) of Saabas and Baliga [15] is adapted to the standard staggered grid in which pressure and velocity components are stored at different points. The SIMPLER algorithm was applied to resolve the pressure–velocity coupling in conjunction with an Alternating Direction Implicit

(ADI) scheme for performing the time evolution. The numerical code used here is described and validated in details in Abbassi and Turki [16, 17]. From the known temperature and velocity fields at any instant τ given by solving (1–4), the local entropy generation $S_{l,a}$ is evaluated at any node of the domain by (8). The dimensionless total entropy generation for the entire channel S_t is obtained by (12). The evolution of S for many periods permits the evaluation of the time averaged entropy generation $\langle S_t \rangle$ by using (13).

The averaged Nusselt number at the top wall is used for the grid independence analysis. Grid refinement tests have been performed for the case $Re = 10$, $Pe = 20/3$, $\varepsilon = 0.5$, $Da = 10^{-2}$ and $Br^* = 10^{-4}$ using three uniform grids 70×20 , 101×26 and 131×31 . Results show that the relative error is equal to 4.24 % when we pass from the grid of 70×20 to the grid of 101×26 , and it becomes 1.74 % when we pass from the grid of 101×26 to the grid of 131×31 . We conclude that the grid 101×26 is sufficient to carry out a numerical study of this flow. This grid is retained for all following investigations. The imposed global and local convergence criteria are given, respectively, by:

$$\left(\frac{\partial V_x}{\partial X} + \frac{\partial V_y}{\partial Y} \right) \leq 10^{-5}, \quad \max \left| \frac{\Gamma^{\tau+\Delta\tau} - \Gamma^\tau}{\Gamma^{\tau+\Delta\tau}} \right| \leq 10^{-5} \quad (16)$$

Where I is the dependent variable, $\Gamma = (V_x, V_y, \theta)$.

5. Results and discussions

In this investigation, the Reynolds and the Prandtl numbers are fixed at 10 and 0.7 respectively. In this investigation, the Darcy, Brinkman and Rayleigh numbers were fixed respectively at $Da = 1$, $Br^* = 10^{-3}$ and $Ra = 10^4$. The porosity parameter is varying between 0 and 1. The viscosity ratio and the specific heat capacity ratio they are fixed to unity.

Let's start with the case where the porosity is equal to unity. The Darcy, the modified Brinkman and the Rayleigh numbers are equal to 1, 10^{-3} and 10^4

respectively. Fig.2 illustrates the evolution of the total entropy generation with dimensionless time.

As can be seen from this figure, the total entropy generation evolution is periodic but not sinusoidal, with two maximums and two minimums, both absolute and relative extremes are oscillating around average value of $\langle S_t \rangle = 10.86$ with dimensionless period $\Theta = 2.6$. This is due to the fact that total entropy generation is the sum of many temperature and velocity gradients having different phases and amplitudes. This periodic behavior of the total entropy generation indicates the existence of the Poiseuille–Benard flow configuration, characterized by the presence of thermo-convective cells near the bottom and the top walls as illustrated by streamlines plotted in Fig.4. The convective cells appear in alternation near the bottom and the top walls moving in the direction of the main flow as cylinders turning without translation on walls. The cells on the bottom are turned clockwise whereas the cells of the top wall turn in the anti-clockwise direction. In thermodynamics of irreversible processes, this configuration maintained by energy dissipation is known as dissipative structure. In a thermodynamic view point, this case corresponds to a rotation of the system around the steady state, which is far from equilibrium one, and consequently the system is in the nonlinear domain of the thermodynamics of irreversible processes, since the Prigogine’s theorem of minimum entropy generation is unverified. In the following, we have tried to investigate the porosity effect on the presence of dissipative structures in the channel flow. Results show that these structures, whose existence is proven by the periodic fluctuation of the total entropy generation, are maintained up to the porosity value 0.2. One can notice that, although the periodic behaviour of entropy generation persists, the time-averaged entropy generation and its period of oscillations change by decreasing the porosity. At critical porosity number equal to 0.2, a significant change of the total entropy generation occurs. As can be seen in Fig.3, entropy generation oscillates with an

important initial value (not seen in this figure) at the very beginning of mixed convection, then it decreases as dimensionless time proceeds with pseudo-periodic form and reaches a constant value close to 4.90, corresponding to the steady state of the mixed convection.

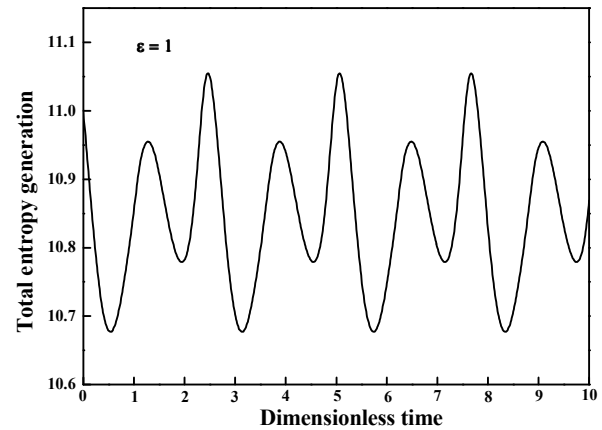


Fig. 2 Variation of the total entropy generation as a function of dimensionless time at $\varepsilon = 1$

Again the system evolves in the non-linear branch of irreversible thermodynamics, except that the system, this time, performs a spiral approach towards the steady state. The time limited fluctuations of the total entropy generation leads to believe at the birth of thermo-convective cells that are rapidly lost as time increases. Indeed the existence of these thermo-convective cells is highlighted by the plot of the streamlines at different dimensionless times (Fig.5). As can be seen from these plots related to the figure 5, three cells appear in the channel at the very beginning of the fluid motion. Those cells rotate and move toward the end of the channel. It’s noteworthy that their number decreases until their disappearance, practically at dimensionless time equal 10, indicating the stratification of the streamlines and consequently the completion of the convective heat transfer. At local level, the entropy generation distribution in the channel is plotted in Fig.6. It can be seen, that entropy generation is confined near the bottom and the top walls. No significant entropy generation is seen in the

central flow. When the porosity decreases the maximum entropy generation remains localized at the bottom and the top walls, except the fact that the length of the active walls not engendered by energy dissipation increases. It should be noted that for porosity less than 0.4, the space not induced by entropy generation increases, simultaneously isentropic lines become increasingly confined on the active walls of the channel.

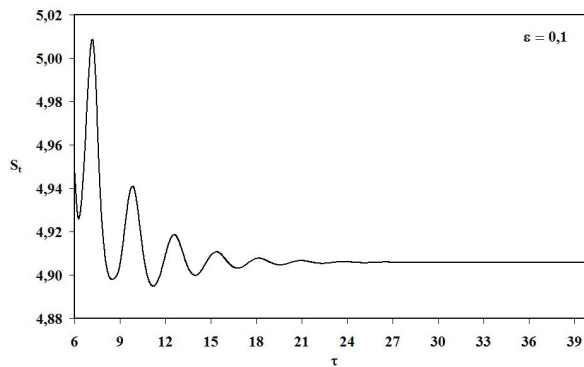


Fig. 3 Variation of the total entropy generation as a function of dimensionless time at $\epsilon = 0.1$

At relatively small porosity number local entropy generation are insignificant and practically absent in the channel. This can be explaining by the fact that when porosity decreases the fluid friction diminishes in the medium, inducing a reduction of the viscous flow dissipation (conductive heat transfer dissipation dominates) and consequently a decrease of the total entropy generation.

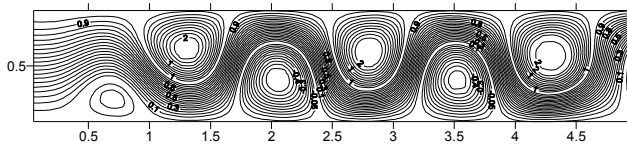


Fig.4 Stream lines at $\epsilon = 1$

6. Conclusions

The investigation of entropy generation in 2D laminar porous channel flow has been studied numerically from direct solutions of complete Navier–Stokes and energy equations. Reynolds and

Peclet numbers were fixed at $Re = 10$ and $Pe = 20/3$. Darcy number, Rayleigh number and Brinkman number were fixed at $Da = 1$, $Ra = 10^4$ and $Br^* = 10^{-3}$ respectively. Results show the existence of dissipative structures, which are maintained in the channel when porosity varies from 1 to 0.2. This case corresponds, in a view point of thermodynamics of irreversible processes, to a rotation of the system around the steady state. These dissipative structures disappear under porosity equal to 0.2 and the total entropy generation oscillates with pseudo-periodic regime before reaching the steady state corresponding to a constant value of entropy generation. This case corresponds to a spiral approach towards the steady state. In the two cases the system evolves in the nonlinear branch of the thermodynamic of irreversible processes. At local level, entropy generation is localized just near the active walls of the channel. The space not induced by entropy generation increases when the porosity number decreases.

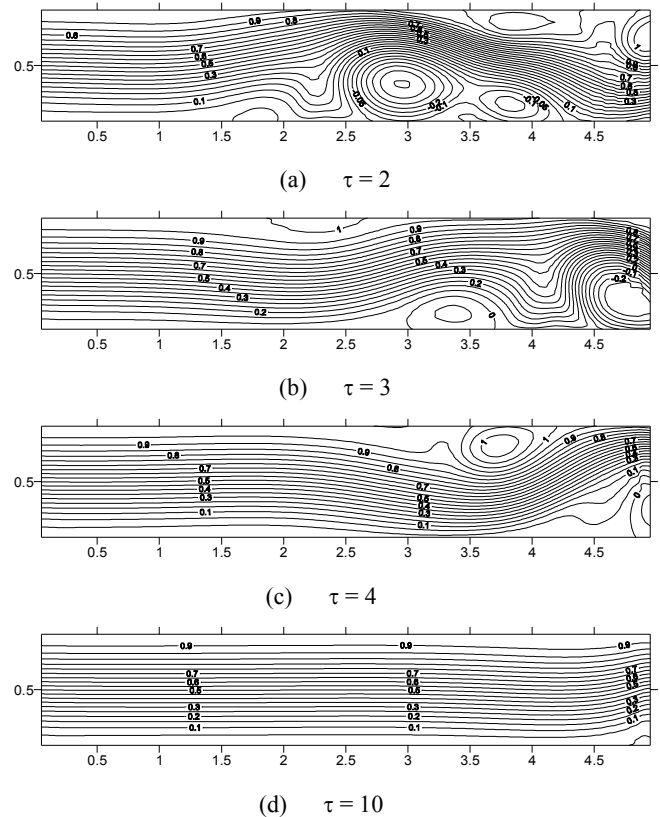


Fig. 5 Evolution during time of stream lines at $\epsilon = 0.1$

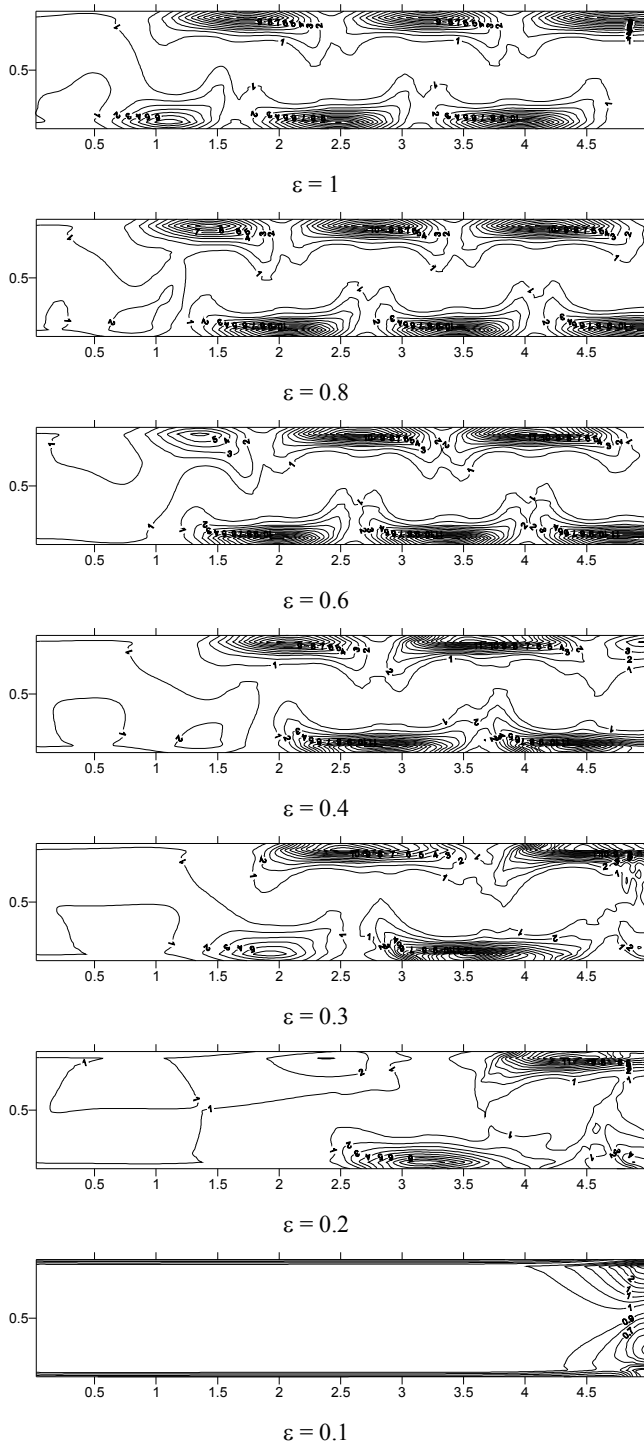


Fig. 6 Evolution of isentropic lines for different values of porosity

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