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# Water hammer wave for leak detection in elastic and viscoelastic pipes

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**Abstract:** This paper further investigates leaks detection in elastic and viscoelastic pipes based on the analysis of water hammer wave. Leak as an internal exciter and a discontinuity that occurs in a hydraulic system may affect the waveform. Its effects on the pressure wave at the end section of a reservoir-pipe-valve system can be used as a tool for its location and sizing. The leak modeled as an orifice is located in the studied system either with elastic or viscoelastic behavior. Its location is given by analyzing the head pressure at the valve either in the time or the frequency domain. The effect of the pipe-wall elasticity (instantaneous elastic response) and viscoelasticity (retarded –viscoelastic response) on the leak detection are investigated either in time or frequency domain. Finally a comparison between the two analyses for leak detection is drawn.

**Key words:** Water Hammer, Leak detection, Elastic pipe, viscoelastic pipe, Transient pressure.

## 1. Introduction

The transient response of a hydraulic system given by the head pressure fluctuation at one section of the pipeline system after the sudden closure of a valve is used as a tool for detecting hydraulic singularities (leakage, blockage…). This tool needs an interesting knowledge of the physical fluid and pipeline system proprieties and their interaction. This knowledge is the key to successful management of pipeline operations and control. Enormous complexities of fluid structure interaction are predicted by combining physical laws, mathematical abstraction and numerical procedures. Leak detection is a well studied area, a number of hydraulic transient-based techniques for locating and sizing leaks are described in literatures and they vary from the simple physical control to acoustic method.

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Most of these techniques are until now widely used such as Ground penetration radar.

These techniques search to give a clear picture of the internal conditions of the pipelines, so and the Transient Analysis is a promising development. The behavior of the wave propagating along the pipeline system depends on the fluid properties and the type of the pipe material (structure). The leak as a material discontinuity and hydraulic singularity considerably modify this wave.

Since the beginning of twentieth century numerous transient methods have been developed to localize leak.

These different methods are based on the comparison of pressure wave of a pipeline system with or without leak. The effect of a leak on the transient generated by the sudden closure of the valve appears in pressure signal at the valve boundary of a reservoir –pipe-valve system.

The analysis of the transient pressure signal can be ran either in time or frequency domain. Several works have been investigated the leak detection by time domain analysis of the transient pressure wave: in [15], the leak was located from the arrival time of the reflected wave, in [9] leak is located and sized based on the head pressure drop due to the leak reflection, in [3] the time difference between the wave reflected by the leak and that at the valve section was used for leak location in viscoelastic pipe.

In frequency domain the leak was sized and located using frequency analysis method based on: the head pressure analysis [13], the Wavelet analysis [14], the frequency response method [12], the Standing wave difference method [11] and the impedance method [2].

All previous works have shown that the leak signature, appearing when analyzing the pressure wave, depends on several hydraulic parameters of the studied system and local characteristics of the leak.

The aim of this paper is decoding the effect of several hydraulic parameters on the leak location and sizing. This is first studied by the time-domain analysis of the head pressure at the end section of the well studied system (reservoir-pipe-valve), secondary by the frequency domain analysis based on the fast Fourier transform of the head pressure at the valve. Either elastic or viscoelastic (reservoir-pipe-valve) system with one leak are investigated.

## 2. Model development.

Hydraulic transient in closed pipes have been a subject of both theoretical study and practical investigation for more than one century. Unsteady flow has been studied since man first bent. In this section two models are developed to describe the unsteady state flow respectively in elastic and viscoelastic pipes.

*2.1 Linear elastic model*

The simplified one-dimensional continuity and momentum equations that describe transient flow in elastic pipe are (Wylie et al 1993):

$$\frac{dH}{dt}+\frac{C^2}{gA}\frac{\partial Q}{\partial x}=0 \tag{1}$$

$$\frac{\partial H}{\partial x}+\frac{1}{gA}\frac{dQ}{dt}+\frac{\lambda Q|Q|}{2gDA}=0 \tag{2}$$

where H is the pressure head, Q is the flow rate, C is the elastic wave speed, g is the gravitational acceleration, A is the pipe cross-sectional area, D is the internal pipe diameter, $\lambda$ is the coefficient of friction, x is the special coordinate and t is the time.

The elastic wave speed, *C*, is a parameter that depends on the fluid compressibility and on the physical properties and external constraints of the pipe. Assuming linear-elastic behaviour of the pipe wall (described by Hooke's law), wave speed can be estimated by [10,11].

$$C=\left(\rho\left(\frac{1}{K}+\left(1-\nu^2\right)\frac{D}{eE}\right)\right)^{-1/2} \tag{3}$$

where E is the Young's modulus of elasticity of the pipe, K is the fluid bulk modulus, $\rho$ is the fluid density, e is the pipe wall thickness and c is the pipe constraint factor.

*2.2 Linear viscoelastic model*

Viscoelastic pipes have different behavior in comparison with metal and concrete pipes. In this case, strain can be decomposed into an instantaneous elastic strain $\varepsilon_\phi^e$, and retarded strain $\varepsilon_\phi^r$

Taking into account the relationship between the pipe cross-section $A$ and the total circumferential strain, the elastic strain, the retarded strain and the state equation of a barotropic fluid, the equations system (4and 5) becomes [3, 10].

$$\frac{dH}{dt}+\frac{C^2}{gA}\frac{\partial Q}{\partial x}+\frac{2C^2}{g}\frac{d\varepsilon_\phi^r}{dt}=0 \tag{4}$$

$$g\frac{\partial H}{\partial x}+\frac{1}{A}\frac{dQ}{dt}+\frac{g\lambda Q|Q|}{2DA^2}=0 \tag{5}$$

The retarded circumferential strain is represented by a generalized Kelvin-Voigt model of three elements

$$\varepsilon_{\phi}^{r} = \sum_{j=1}^{3} \varepsilon_{\phi,j}^{r}(x,t) = \sum_{j=1}^{3} \left\{ \frac{cD}{2e} \int_0^t \begin{bmatrix} p\left(x,t-t'\right) \\ -p_0(x) \end{bmatrix} \times \frac{J_j}{\tau_j} e^{-t'/\tau_j} dt \right\} \quad (6)$$

where c: parameter characterizing the type of anchoring the pipe and J: the creep compliance function described by [3]:

$$J(t) = J_0 + \sum_{j=1}^{3} J_j \left(1 - e^{-t/\tau_j}\right) \quad (7)$$

in which $J_0$ represents the creep-compliance of the first spring, $J_j$ is the creep-compliance of the spring of j-element and $\tau_j$ is the retardation time of the dashpot of j-element.

The retarded strain time-derivative in equation (4) is calculated by:

$$\frac{\partial \varepsilon_{\varphi}^{r}}{\partial t} = \sum_{j=1}^{n} \frac{\partial \varepsilon_{\phi,j}^{r}(x,t)}{\partial t} = \sum_{j=1}^{n} \frac{cD}{2e} \frac{J_j}{\tau_j} \times [p(x,t) - p_0(x)] - \frac{\varepsilon_{\phi,j}^{r}(x,t)}{\tau_j} \quad (8)$$

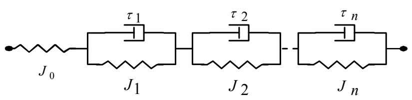



**Fig.1 Generalized Kelvin-Voigt model**

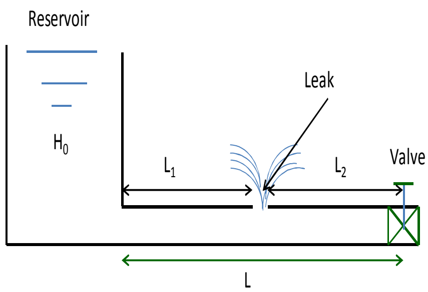



**Fig.2 Reservoir-pipe-valve system with leak**

*2.3 Leak modeling*

Leak represent fault that pipeline systems can experience during their design lifetime, it's modeled using the orifice equation [6, 1, 3].

$$Q_\ell = C_d A_\ell \sqrt{\frac{2(p_\ell - p_0)}{\rho}} \quad (9)$$

where $C_d$ is a discharge coefficient, $A_\ell$ is the orifice area and $p_\ell$ is the head on either side of the orifice assumed to be equivalent.

*2.4 Numerical scheme*

Equations (4) and (5) can be transformed into a system of ordinary differential equations and solved by the method of characteristics (MOC). The compatibility equations are:

$$Q_{Pi} - Q_{i+1} - \frac{gA}{C}\left(H_{Pi} - H_{i+1}\right) + \frac{\lambda}{2DA} Q_{i+1}\left|Q_{i+1}\right| \Delta t + 2CA\Delta t \left(\frac{\partial \varepsilon_r^{\phi}}{\partial t}\right)_{i+1} = 0 \quad (10)$$

along the positive characteristic line $\left(\frac{dx}{dt} = +C\right)$ and

$$Q_{Pi} - Q_{i+1} - \frac{gA}{C}\left(H_{Pi} - H_{i+1}\right) + \frac{\lambda}{2DA} Q_{i+1}\left|Q_{i+1}\right| \Delta t - 2CA\Delta t \left(\frac{\partial \varepsilon_r^{\phi}}{\partial t}\right)_{i+1} = 0 \quad (11)$$

along the negative characteristic line $(dx/dt = -C)$

where $i$ is the node number, $dx$ and $dt$ are the distance and the time steps respectively .

A grid of characteristics is now established in order to accomplish an orderly computer solution. The pipe of length L is initially subdivided into two segments of pipes every part of pipe have a length $L_k$ discritized into equal $N_k$ space-steps $\Delta x = L_k / N_k$ . The leak is introducing as a boundary condition (9) .

*2.5 Initial conditions*

In this paper attention is focused mainly on transients occurring in a single pipe with a constant pressure reservoir at the upstream end and a rapid closure valve at the downstream end (fig. 2). One leak is supposed to exist at an intermediate section of the pipe and located at the distances $L_1$ from the reservoir. The pipe with a length $L$ is subdivided into two segments, first segment of pipe from the reservoir to the leak, the second segment of pipe from the leak to the valve (fig.2).

In order to solve the problem, initial conditions must be provided at the time 0. These conditions can be determined by computing the solution of the following system of ordinary differential equations deduced from (4) and (5):

$$\frac{dQ_k}{dx} = 0 \tag{12}$$

$$\frac{dH_k}{dx} = -\frac{\lambda Q_k^2}{2gDA^2} \tag{13}$$

where $K$=1,

The solution of this system of equations (12) and (13) is given by (14) and (15):

$$Q_k(0,x) = Q_k(0,0) \tag{14}$$

$$H_k(0,x) = H_k(0,0) - \frac{\lambda Q_k^2}{2gDA^2} x \tag{15}$$

At the leak: $Q_k(0, L-X) = Q_{k+1}(0,0) + Q_{\ell k0}$ (16)

*2.6 Boundary conditions*

Transient flow is created by the fast closure of the valve at the downstream end ( $x = L$ ). At this section $Q_J(t, X) = 0$ . At the upstream end $x = 0$ and $t > 0$, the condition is given by the reservoir at fixed level $H_1(t,0) = H_0$ .

At the leak, (9) is implemented in the MOC as an internal boundary condition. The head pressure at the leak section is given by (17) [1, 2, 4, and 11].

$$H_P = \left[ \left( -C_d A_\ell \sqrt{2g} + \sqrt{\Delta} \right) \Big/ \left( 4 \frac{gA}{C} \right) \right]^2 \tag{17}$$

$$\begin{aligned} \Delta = & \left( C_d A_\ell \sqrt{2g} \right)^2 - 8 \frac{gA}{C} \Big[ \left( Q_{2,2} - Q_{1,N_1} \right) \\ & - \frac{\lambda \Delta t}{2DA} \left( Q_{2,2} \left| Q_{2,2} \right| - Q_{1,N_1} \left| Q_{1,N_1} \right| \right) \\ & - \frac{gA}{C} \left( H_{2,2} + H_{1,N_1} \right) + 2CA\Delta t \left( \left( \frac{\partial \varepsilon_r^\phi}{\partial t} \right)_{1,N1} + \left( \frac{\partial \varepsilon_r^\phi}{\partial t} \right)_{2,2} \right) \Big] \end{aligned} \tag{18}$$

*For elastic pipe one can obtain the numerical scheme by making the retarded strain equal to zero in all precedent equations.*

## 3. Time Domain Analysis of Water Hammer Wave for Leak Detection

The pipeline is with an elastic linear behavior and with 50.6 mm inner diameter; 6.2 mm wall thickness and the total length of pipeline is 277 m (length between the reservoir and the global valve). Numerical results are calculated for the initial flow of $\left( Q_0 = 1.008 l s^{-1} \right)$ and for a leak located at intermediate section. The transient event is caused by the sudden closure of the upstream valve. Elastic wave speed is an important parameter that defines elastic behavior of the pipeline system by the presence of Young's modulus (3).

It is represented in Fig.3.a) a theoretical example of a pressure surge at the valve section for an instantaneous closure and for different locations of the leak having the same leak discharge $\left(Q_{\ell 0} = 0.1 * Q_0\right)$.

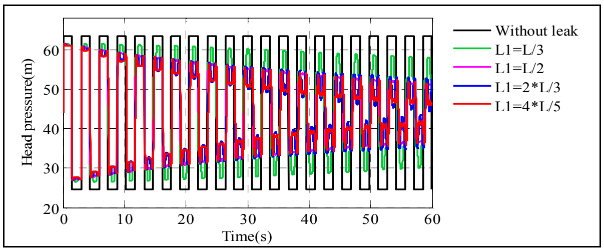
Analyzing fig.3a) one can observe the increase the amplitude of the pressure drop with the decrease distance of the leak from the valve. The leak effect on the water hammer wave can be easily shown in the enlargement of fig.3a) (fig.3.**b)**). This figure allows the leaks location basing on the following equation:

$$X = \left(1 - \frac{\Delta t_{\ell 1}}{t_0}\right) \times L \qquad (19)$$

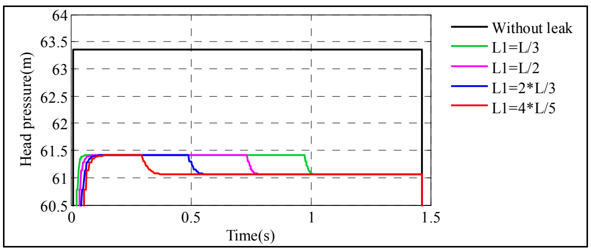
where $\Delta t_{\ell 1}$ is the time difference between the initial transient wave and the reflected wave at the leak section , $t_0 = 2L/C$ is the pipe characteristic time and $X$ is the leak location from the upstream end. Table 1 shows the leak location in the time domain either for elastic pipeline system or viscoelastic one.

The uncertainty of leak location given in the same table by applying (20) [1, 6]

$$\varepsilon_{lock} = \frac{\overline{\overline{C\Delta t_{\ell k}}} - C\Delta t_{\ell k}}{\overline{C\Delta t_{\ell k}}} \quad where\, k = 1, is\ the\ leak\ order \qquad (20)$$

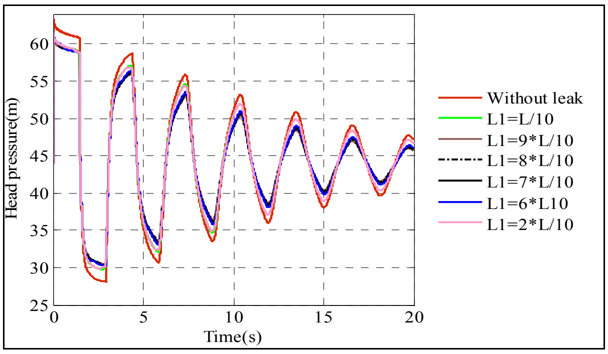



**a)**

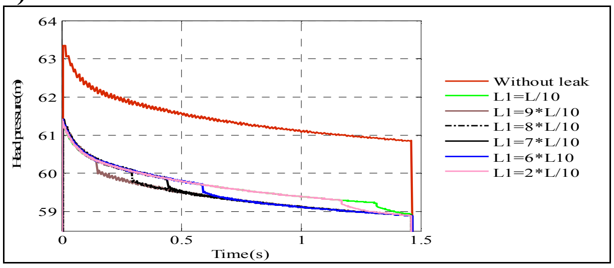



**b)**

**Fig. 3 Head pressure time-evolution at the end section of the reservoir pipe-valve elastic system with leak.**




**a)**




**b)**

**Fig. 4 Head pressure time-evolution at the end section of the reservoir pipe-valve Viscoelastic system**

The same pipeline system is now considered with viscoelastic behavior. The leak effects are drawn in fig.4.a) and its enlargement fig.4.b). The pipeline rheological behaviour linear viscoelastic is described by the creep compliance function (7) represented by three-element Kelvin-Voigt model. Parameters $J_k$ and $\tau_k$ are [3]:

$$J_0 = 0.7E - 9\,Pa^{-1}s, J_1 = 0.0805E - 9\,Pa^{-1}s,$$
$$J_2 = 0.1083E - 9Pa^{-1}s,$$
$$J_3 = 0.57635Pa^{-1}s, \tau_1 = 0.05s, \tau_2 = 0.5s, \tau_3 = 10s.$$

The leak location is determined referring to (19). But one can observe that for viscoelastic pipe, only for the first period the (19) can be applied (fig.4.a) whereas for elastic one this equation can be needed for leak location all the time of valve pressure measurement this is due to the retardations times of the dashpots figuring in the Kelvin Voight model.

## 4. Frequency Domain Analysis of Water Hammer Wave for Leak Detection

In order to locate leak by frequency domain analysis a fast Fourier transform is applied to the different pressure wave evolution in time domain of the previous section and in the same conditions of flow monitoring.

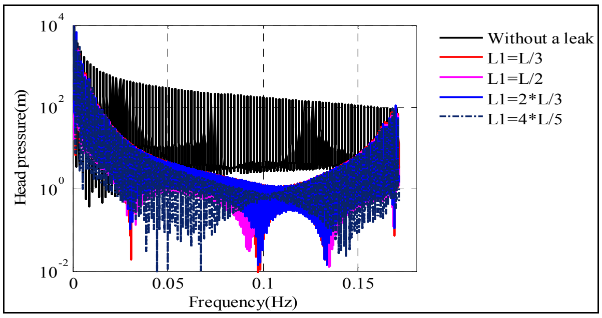
The impact of a leak on the frequency response of a single elastic pipeline system is a well researched area but not for viscoelastic one. It was found for a single elastic pipe system that leak imposes a periodic pattern onto the resonant peaks of the response. The analytical expression relating the leak properties and its imposed pattern on the head pressure at the valve, was first derived in [16] in addition an empirical relation was given in [2].

$$X = \left( \frac{\Delta f_{K,K+1}}{\Delta f_{Intact\ pipe}} \right) \times L \quad (21)$$

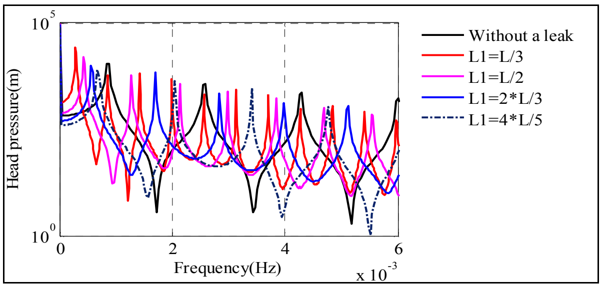
where:

$\Delta f_{K,K+1}$ is the band frequency width between two successive-resonant-peaks (damaged pipe).

$\Delta f_{Intact\ pipe}$ is the band frequency width between two consecutive-resonant-peaks of high amplitude (intact pipe).



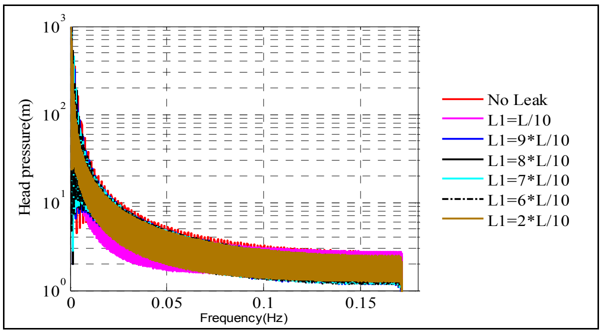

**a)**



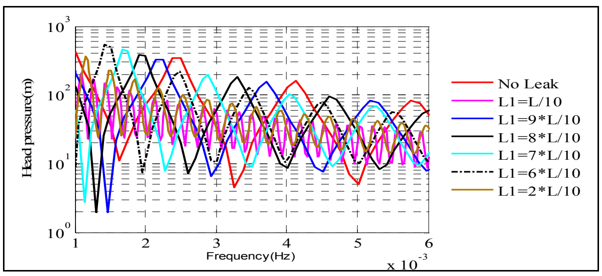

**b)**

**Fig. 5 Head pressure frequency-evolution at the end section of the reservoir pipe-valve elastic system.**

Figs. 5.b and 6.b) represent respectively the enlargement of figs. 5.a) and 6.a) one can locate the leak by pressure frequency domain analysis of these two figs. Either in elastic pipe system or viscoelastic one $\Delta f_{K,K+1}$ can be measured but for a viscoelastic system there is no resonant peaks so,and the band frequency width of leaking pipe is considered between two consecutive maximums of the head pressure amplitude and not between two resonant peaks.




a)




b)

**Fig. 6 Head pressure frequency-evolution at the end section of the reservoir pipe-valve Viscoelastic system**

**Table 1 Leak locations in time domain**

| System behavior | $t_0[s]$ | $t_{\ell 1}[s]$ | $X_1(Actual\ value)[m]$ | $X_1(Simulate\ value)[m]$ | $\varepsilon_{\ell oc}(\%)$ |
|---|---|---|---|---|---|
| Elastic-system | 1.463 | 0.9755 | 91.41 | 92.301 | 0.9 |
| | 1.463 | 0.7316 | 138.5 | 138.5 | 0 |
| | 1.463 | 0.487 | 182.82 | 184.79 | 0.3 |
| | 1.463 | 0.2926 | 221.6 | 221.6 | 0 |
| Viscoelastic system | 1.463 | 1.315 | 27.7 | 28.021 | 1.15 |
| | 1.463 | 0.1478 | 249.3 | 249.016 | 0.1 |
| | 1.463 | 0.2941 | 221.6 | 221.31 | 0.13 |
| | 1.463 | 0.4404 | 193.9 | 193.61 | 0.146 |
| | 1.463 | 0.5882 | 166.2 | 165.63 | 0.34 |
| | 1.463 | 1.168 | 55.4 | 55.85 | 0.82 |

**Table 2 Leak locations in frequency domain**

| System behavior | $\Delta f_{int\ act\ pipe}$ | $\Delta f_{K,K+1}$ | $X_1(Actual\ value)$ | $X_1(Simulate\ value)$ | $\varepsilon_{\ell oc}(\%)$ |
|---|---|---|---|---|---|
| Elastic-system | 0.001712 | 0.00057 | 91.41 | 92.3795 | 1 |
| | 0.001712 | 0.000856 | 138.5 | 138.5 | 0 |
| | 0.001712 | 0.001141 | 182.82 | 184.612 | 0.9 |
| | 0.001712 | 0.001369 | 221.6 | 221.323 | 0.125 |
| Viscoelastic system | 0.001629 | 0.000166 | 27.7 | 28.227 | 1.9 |
| | 0.001629 | 0.001578 | 249.3 | 268.327 | 7.7 |
| | 0.001629 | 0.001403 | 221.6 | 238.4 | 7.63 |
| | 0.001629 | 0.001227 | 193.9 | 208.64 | 7.59 |
| | 0.001629 | 0.001070 | 166.2 | 177.834 | 7 |
| | 0.001629 | 0.000666 | 55.4 | 52.203 | 5.7 |

Table 2 indicates the validity of (21) to locate the leak in elastic or viscoelastic pipe by analyzing the frequency-head pressure evolution. It can be noticed that the leak location uncertainty is higher for viscoelastic system, this is due to the water hammer wave attenuation in shape and timing (retarded strain).

The leak location in time domain is more rapid and has low time consuming.

## 6. Conclusions

Transient in pipeline are modified by singularities such as leakage, this allows to investigate the leak detection either in time or frequency domain basing on the leaks effects on water hammer wave at the end section of a reservoir-pipe-valve system.

Two procedures were illustrated numerically to locate leak in elastic and viscoelastic pipeline system. The first is based on the effect of the wave reflected by the leak on wave pressure time evolution, simulated at the valve. The analysis of this effect allows the leak location and shows that its uncertainty is less than $1\%$ for elastic pipeline system and $2\%$ for viscoelastic one. The second is the frequency domain leak detection techniques, based on the properties of leak-induced pattern-leading to the leak location. It was concluded that the first one is more promising for elastic system than viscoelastic one, yet the leak locations results are acceptable in two cases. The second method has an uncertainty that can reach $8\%$, it can be a time consuming if the leak is closer to the valve because of the occurrence of multiple reflection of water hammer wave before its return to the valve and to be properly registered (measured).

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