International Journal of Mechanics and Energy (IJME)
Vol. 1, Issue 4, 2013, ISSN: 2286-5845

Evaluation of the models of the pressure-strain correlation in the turbulent compressible flow

Aicha Hanafi *, Hachmi Khlifi, Taieb Lili

Faculty of Sciences of Tunis, Elmanar 1060, Tunis, Tunisia

Laboratory of Fluid Mechanics (LFM)

Abstract: Several DNS results show that compressibility has an important effect on the pressure-strain correlation, the term recognized as the principal responsible for the change in the magnitude of Reynolds-stress anisotropies. Thus, the pressure-strain incompressible models do not correctly predict compressible turbulence at high-speed shear flow. A method of including compressibility effects in the pressure strain correlation is the subject of the present study. The LRR model developed by Launder-Reece and Rodi has shown a great success in the simulating a variety of incompressible complex turbulent flows. On the other hand this model has not predicted correctly the compressible turbulence at high speed shear flow. Thus, a compressible correction for this model is the major aim of this study. In the present work, five recent compressible models for the pressure-strain correlation have been used to modify the LRR model. This correction concerns essentially the C1, C2, C3 and C4 coefficients which became in a compressible situation a function of the turbulent Mach number.

Key words: Compressible turbulence, Pressure-strain correlation, Homogeneous shear flow .

1. Introduction

The comprehension of compressibility effects on turbulence is fundamental for many industrial applications, such as combustion, environment and aerodynamics. These effects became significant when the mean flow is strongly deformed or when the turbulent kinetic energy contained in the dilatational fluctuations is important. It is well known that the growth rate of turbulent kinetic energy is critically reduced with increasing turbulent Mach number. In this context, many studies of the compressible shear flow show the changes of the turbulence structures are principally due to the structural compressibility effects which significantly affect the pressure-strain correlation.

Eventually, the pressure-strain correlation appears as the main factor for the changes in the magnitude of the Reynolds stress anisotropies. The extension of the standard models to compressible flows represents a research topic of great scientific and industrial interests. As in incompressible turbulent situation the experimental and numerical results are used to examine the performance of the corrections proposed by some authors to the LRR model for the pressure-strain correlation, thus the subject of this paper is to analyze the turbulence models developed to study compressible flows and to evaluate its performance with DNS results developed by Sarkar et al.[8] in uniformly compressible shear flow.

1.1. Governing equations

Prepare The General equations governing the motion of a compressible fluid are the Navier-Stokes equations. They can be written as follows for mass, momentum and energy conservation:

$$\frac{\partial \rho}{\partial t}+\frac{\partial\left(\rho u_i\right)}{\partial x_i}=0 \tag{1}$$

$$\frac{\partial(\rho u_i)}{\partial t}+\frac{\partial(\rho u_i u_j)}{\partial x_j}=-\frac{\partial p}{\partial x_i}+\frac{\partial \sigma_{ij}}{\partial x_j} \tag{2}$$

* **Corresponding author:** Aicha Hanafi
E-mail: aicha.hanafi@yahoo.fr

$$\frac{\partial}{\partial t}(\rho E)+\frac{\partial}{\partial x_j}(\rho E u_j)=\frac{\partial}{\partial x_j}(-pu_j+\tau_{ij}u_i)-\frac{\partial}{\partial x_j}\left(-k\frac{\partial T}{\partial x_j}\right) \quad (3)$$

where

$$\sigma_{ij}=\mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}-\frac{2}{3}\frac{\partial u_i}{\partial x_i}\delta_{ij}\right),\tau_{ij}=\mu(u_{i,j}+u_{j,i})-\frac{2}{3}\mu u_{k,k}\delta_{ij},E=e+(u_i u_i/2)$$

and $e=C_{vT}$

For an ideal gas, the relation between pressure, density and temperature can be written as follows:

$$p=\rho RT \quad (4)$$

1.2 Averaged equations:

In incompressible flows, two averaging techniques can be used to split a physical quantity F, into an averaged and a fluctuating term. Such techniques are known as the ensemble and the mass-weighted averages, which are often referred to as the Reynolds and the Favre averages respectively. For the Reynolds average technique, F is divided into a mean part, $\overline{F}$, and a fluctuating part, F', as

$$F(x_i,t)=\overline{F}(x_i,t)+F'(x_i,t),\quad \overline{F'}(x_i,t)=0$$

While in the Favre average, except density and pressure, the quantity F is written in the following form

$$F(x_i,t)=\tilde{F}(x_i,t)+F''(x_i,t)$$

Where the Favre mean is defined as

$$\tilde{F}(x_i,t)=\overline{\rho F(x_i,t)}/\overline{\rho}$$

$$\overline{\rho F''}(x_i,t)=0,\quad \overline{F''}(x_i,t)=-\overline{\rho' F''}(x_i,t)/\overline{\rho}\neq 0$$

And the Favre fluctuation F'' satisfies the following relationships:

1.3. basic equations of the Favre second-order closure in compressible homogeneous turbulent shear flow

For compressible homogeneous shear flow, the mean velocity gradient is given by:

$$\tilde{U}_{i,j}=S\delta_{i1}\delta_{j2}=\begin{cases} S\, si(i,j)=(1,2) \\ 0, i\neq 1, j\neq 2\end{cases} \quad (5)$$

Where S is the constant mean shear rate. These considerations lead to: $\tilde{U}_{k,k}=0,\quad \overline{\rho}=cte$

At high Reynolds numbers, when assuming the hypothesis of isotropy dissipative structures of the turbulence, the Favre averaged Reynolds stress should be a solution of the transport equation:

$$\overline{\rho}\frac{d}{dt}\left(\widetilde{u_i'' u_j''}\right)=P_{ij}+\varphi_{ij}^{*}+\frac{2}{3}\pi_d\delta_{ij}-\frac{2}{3}\overline{\rho}(\varepsilon_s+\varepsilon_c)\delta_{ij} \quad (6)$$

The governing equation of turbulent kinetic energy, $k=\widetilde{u_i'' u_i''}/2$, is:

$$\overline{\rho}\frac{d}{dt}k=P+\pi_d-\overline{\rho}(\varepsilon_s+\varepsilon_c) \quad (7)$$

Classically, the second-order closure requires a transport equation of the turbulent dissipation rate. The new concept of dissipation in compressible turbulence was proposed by Sarkar et al.[3], Zeeman[2] and Ristorcelli [4] and can be written as follows:

$$\varepsilon=\varepsilon_s+\varepsilon_c \quad (8)$$

$\overline{\rho}\varepsilon_c=\frac{4}{3}\mu\overline{(u'_{i,i})^2}$ represent the solenoidal and compressible parts of ε respectively .Sarkar et al.[3] have mentioned that for moderate Mach numbers, ε_s is insensitive to the compressibility changes. This yields, for ε_s, a model transport equation, similar to what it was obtained in the incompressible case. Such a model equation is written as in [1], namely:

$$\frac{d\varepsilon_s}{dt}=C_{\varepsilon 1}\frac{\varepsilon_s}{k}P-C_{\varepsilon 2}\frac{\varepsilon_s^2}{k} \quad (9)$$

Where $C_{\varepsilon 1}$ and $C_{\varepsilon 2}$ are respectively the model constants $C_{\varepsilon 1}=1.44$ and $C_{\varepsilon 2}=1.83$

We should remind that ε_c is generally taken to be proportional to ε_S through the following algebraic equation: $\varepsilon_c=f(M_t)\varepsilon_s$

$F(M_t)$ is a function of the turbulent Mach number.

As it is suggested in model of sarkar[3], one can write:

$$f\left(M_t\right) = \alpha M_t^2$$

Where α is constant model, α=0.5 in homogeneous turbulence.

Sarkar et al.[3] have also proposed a model for the pressure-dilatation correlation in term of turbulent Mach number as follows

$$\pi_d = -\alpha_2 P M_t + \alpha_3 \bar{\rho}\varepsilon_s M_t^2 \tag{12}$$

Where P the turbulent kinetic energy production: $P = -\bar{\rho}\widetilde{u_i'' u_i''}\tilde{U}_{i,j}$

The model constants α_2, α_3 take the values α_2 =0.15, α_3=0.2

The turbulent Mach number is described by the transport equation as follows

$$\frac{dM_t}{dt} = M_t \frac{P}{2k} + \frac{M_t}{2\bar{\rho}k}\left[1 + \frac{1}{2}\gamma(\gamma - 1)M_t^2\right]\left(\pi_d - \bar{\rho}\varepsilon_s - \bar{\rho}\varepsilon_c\right) \tag{13}$$

2. Model of turbulence :

2.1-Models of the pressure-strain correlation

Model of Launder Reece and Rodi[7].

$$\begin{aligned}\Phi_{ij}^* = &-C_1\bar{\rho}\varepsilon_s b_{ij} + C_2\bar{\rho}k\left(\tilde{S}_{ij} - \frac{1}{3}\tilde{S}_{kk}\delta_{ij}\right) + \\ &C_3\bar{\rho}k\left(b_{ip}\tilde{S}_{jp} + b_{jp}\tilde{S}_{ip} - \frac{2}{3}b_{pq}\tilde{S}_{pq}\delta_{ij}\right) + \\ &C_4\bar{\rho}k\left(b_{ip}\tilde{\Omega}_{jp} + b_{jp}\tilde{\Omega}_{ip}\right)\end{aligned} \tag{14}$$

Where C_1, C_2, C_3 and C_4 are constants that take on the values of: C_1=3, C_2 =0.8 et C_3=1.75 et C_4=1.31

$$\tilde{S}_{ij} = \frac{1}{2}\left(\tilde{u}_{i,j} + \tilde{u}_{j,i}\right), \tilde{\Omega}_{ij} = \frac{1}{2}\left(\tilde{u}_{i,j} - \tilde{u}_{j,i}\right) \quad \text{and} \quad b_{ij} = \frac{1}{2k}\widetilde{u_i'' u_j''} - \frac{1}{3}\delta_{ij}$$

Model of Adumitroaie et al.(1999)

Adumitroaie et al.(1990) assumed that incompressible modeling approach of the pressure strain correlation can be used to develop turbulent models taking into account compressibility effects. Considering a none zero divergence for the velocity fluctuation called the compressibility continuity constraint and using different models for the pressure dilatation which is proportional to the trace of the pressure strain, their model for the linear part of this term is written as:

$$\begin{aligned}\phi_{ij}^* = &-C_1\bar{\rho}\varepsilon a_{ij} + \bar{\rho}k\left(\frac{4}{5} + \frac{2}{5}d_1\right)\tilde{S}_{ij}^* + \\ &\left(1 - C_3 + 2d_2\right)\bar{\rho}k\left[a_{ip}\tilde{S}_{pj}^* + a_{jp}\tilde{S}_{ip}^* - \frac{2}{3}b_{pq}\tilde{S}_{pq}^*\delta_{pq}\right] - \\ &\left(1 - C_4 - 2d_2\right)\bar{\rho}k\left(\left[b_{ip}\tilde{\Omega}_{jp} + b_{jp}\tilde{\Omega}_{ip}\right] + \frac{4}{5}d_2\tilde{S}_{pp}a_{ij}\right)\end{aligned} \tag{15}$$

The compressibility coefficients d_1 and d_2 are determined on the basis of some compressible closures of the pressure-dilatation correlation.

$$d_1 = 0, d_2 = \frac{\alpha_2}{2}M_t, (\alpha_2 = 0.15)$$

Model of Song Fu et al.

The following form for the pressure- strain correlation is proposed as :

$$\begin{aligned}\phi_{ij}^* = &-C_1 a_{ij}\varepsilon + C_2 k(\tilde{S}_{ij} - \frac{1}{3}\tilde{S}_{kk}) + \\ &C_3 k(a_{ij}\tilde{S}_{lj} + a_{jl}\tilde{S}_{li} - \frac{2}{3}a_{lk}\tilde{S}_{kl}\delta_{ij}) \\ &+ C_4 k(a_{lj}\tilde{\Omega}_{il} + a_{il}\tilde{\Omega}_{lj})\end{aligned} \tag{16}$$

$$\text{where} \quad S_{ij} = 0.5(\tilde{U}_{i,j} + \tilde{U}_{j,i}) \quad \text{and} \quad \tilde{\Omega}_{ij} = 0.5(\tilde{U}_{i,j} - \tilde{U}_{j,i})$$

Where C_1 =1.8, C_2 =0.8, C_3=0.6+F(M_t), C_4=0.6-F(M_t)

When F(M_t)=0 and the dilatation of the mean velocity is neglected, the model is the same as the Launder's model. Following the idea of zonal, the effect of the pressure-strain term should quickly increase in the moderate-M_t region. So, it is proposed here as

$F(M_t)=0.25\exp(-0.05/M_t^3)$

Model of MKL[10]:

The contribution of MKL et al. appears in the correction of the C_i coefficients, which became in a compressible turbulence situation a function of the turbulent Mach number. The suggested method is based proportionality relations between the ratio of compressible and incompressible components of the pressure-strain correlation and the ratio relating the compressible and incompressible growth rate of the turbulent kinetic energy. This method generates a pressure-strain model parametrized according to the turbulent Mach number:

$$\phi_{ij}^{*} = -C_1\bar{\rho}\varepsilon_s b_{ij} + C_2\bar{\rho}k\left(\tilde{S}_{ij} - \frac{1}{3}\tilde{S}_{ll}\delta_{ij}\right) +$$
$$C_3\bar{\rho}k\left(b_{ik}\tilde{S}_{jk} + a_{jk}\tilde{S}_{ik} - \frac{2}{3}b_{ml}\tilde{S}_{ml}\delta_{ij}\right)$$
$$+C_4\bar{\rho}k\left(b_{ik}\tilde{\Omega}_{jk} + b_{jk}\tilde{\Omega}_{ik}\right)$$
$$C_1 = C_1^I\frac{\left(1-0.44M_t^2\right)}{\left(1+0.44M_t^2\right)} \qquad (17)$$
$$C_2 = C_2^I$$
$$C_3 = C_3^I\left(1-1.5M_t^2\right)$$
$$C_4 = C_4^I\left(1-0.5M_t\right)$$

***Results and discussion**

The transport the transport equations (6), (7), (9) and(13) on which the second order closure for compressible homogenous shear flow is based, are solved using the forth-order accurate Runge-Kutta numerical scheme. The model predictions will be compared with DNS results conducted by Sarkar **[6]** for cases A_1, A_2, A_3 and A_4.These cases correspond to different initial conditions for which the initial values of the gradient Mach number M_{g0} change by changing the initial values of $(Sk/\varepsilon)_0$ and taking M_{t0} constant as it is listed in Table. For the two cases B_1 and B_2 correspond to different initial condition for which the initial values of the turbulent Mach number M_{t0} change by changing the initial values of $(Sk/\varepsilon)_0$ and taking M_{g0} constant

Table1: Initial condition

case	Mt	Mg	ε_S/Sk
A1	0.4	0.22	1.8
A2	0.4	0.44	3.6
A3	0.4	0.66	5.4
A4	0.4	1.32	10.8
B1	0.4	0.22	5.4
B2	0.4	0.22	3.6

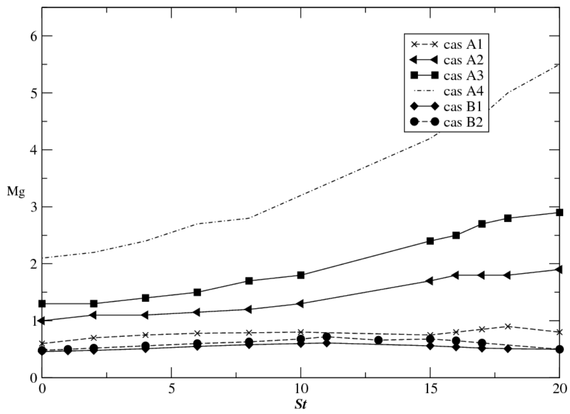

Fig1. The time evolution of the gradient Mach number in cases A and B

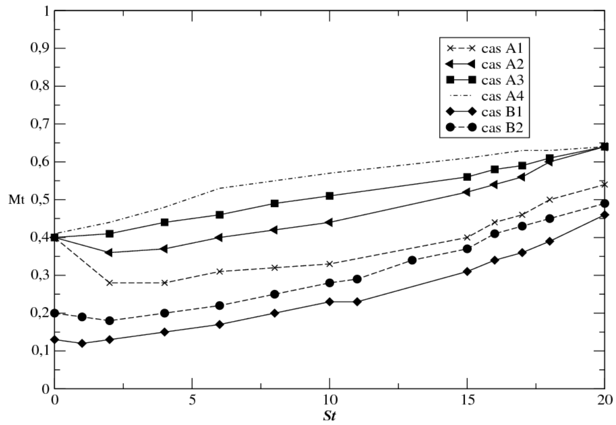

Fig2. The time evolution of the turbulent Mach number in cases A and B

The data of compressible homogeneous shear flow can also be separated into three different regions. The cases in fig.2 come from the DNS of Sarkar [6]. It is easy to see that the difference of the turbulent Mach number (M_t) among cases A_1, B_1 and B_2 is much larger than the difference among cases A_2, A_3 and A_4; but the difference of M_g among cases A_1, B_1 and $B_2$is much smaller than the difference among cases A_2, A_3 and A_4. Paying more attention, we can see that the turbulent mach numbers of cases A_1, B_1 and $B_2$are nearly all under 0.4, and the turbulent Mach numbers of cases A_2, A_3 and A_4 are nearly all over 0.4. It seems that the data can also be put into different regions. When the turbulent Mach number is under 0.4, the change of M_g is small, so the difference of among cases A_1, B_1 and B_2 is small. Taking another look at the cases A_2, A_3 and A_4, we find the decline rates of M_g are all become smaller with increasing turbulent Mach numbers. It seems to tell when the turbulent Mach number is between 0.4 and 0.6, M_g has a quick change; and when the turbulent Mach number is over 0.6, the change rate of M_g become small again. It can be seen again that three different regions exist in the compressible homogeneous shear flow.

From these observations, it is proposed here that there exist three different regions in response to the compressible effects of turbulence: the low compressibility region (M_t<0.4) case A_1, B_1 and B_2, the moderate compressibility region (0.4<M_t<0.6) case A_2 and A_3, the high compressibility region (M_t>0.6). In different regions, the main compressible effect will be different.

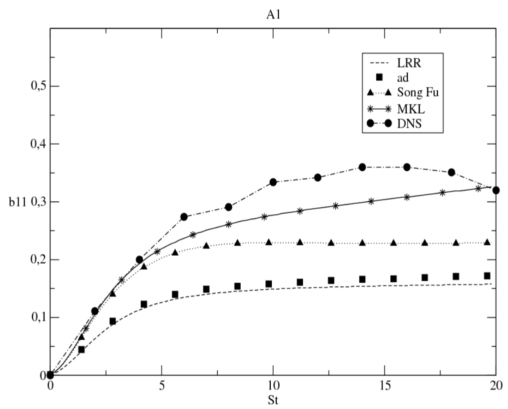

Fig.3a. The time evolution of the Reynolds-stress anisotropy b_{11} in case A1

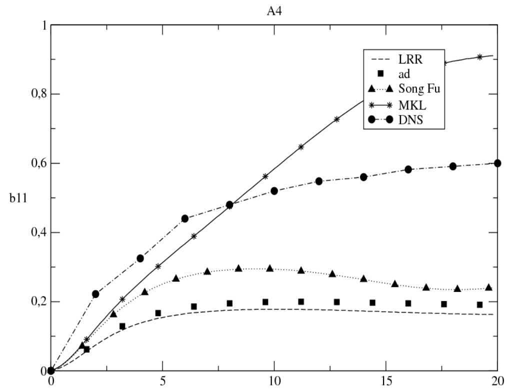

Fig.3b. The time evolution of the Reynolds-stress anisotropy b_{11} in case A4

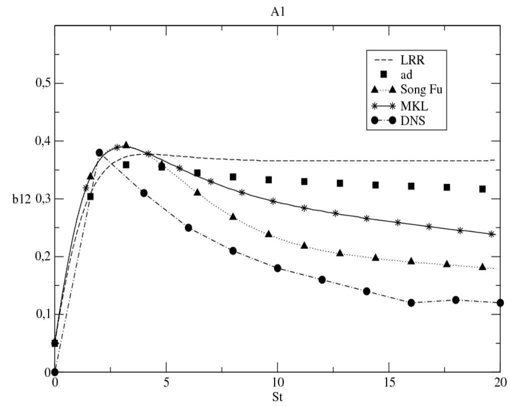

Fig4.a. The time evolution of the Reynolds-stress tensor $-2b_{12}$ in case A1

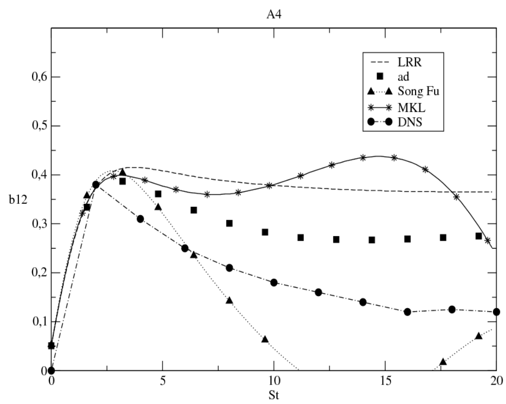

Fig4.b. The time evolution of the Reynolds-stress tensor $-2b_{12}$ in case A4

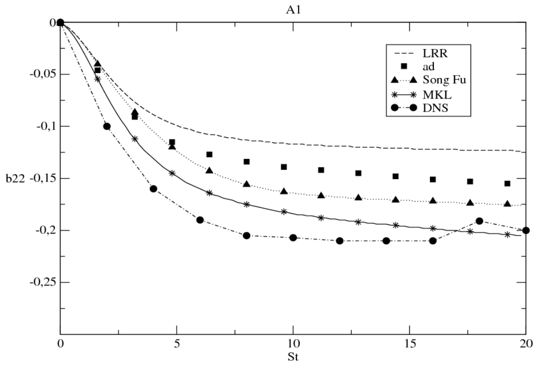

Fig 5.a. The time evolution of the Reynolds-stress tensor b_{22} in case A1

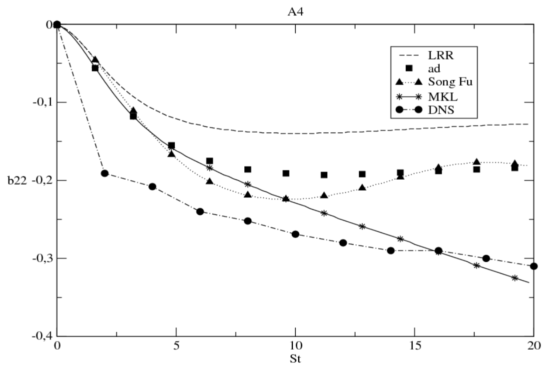

Fig5.b. The time evolution of the Reynolds-stress tensor b_{22} in case A4

From fig3(a, b), fig4(a, b) and fig5(a, b) , it is clear that the incompressible Launder, Reece and Rodi (LRR) model [9] is still unable to predict the dramatic changes in the magnitude of the Reynolds-stress anisotropy and the pressure-strain correlation that arise from compressibility. The time evolution of Reynolds stress-anisotropy shows that the Adumitroaie model yields unacceptable results that are in disagreement with the DNS results. Because the compressibility correction model proposed by MKL et al. contains the turbulent Mach number M_t only, the predicted values for case A_4 are in disagreement with the DNS results. The proposed form of the Song Fu provides an unacceptable performance in reproducing the DNS results for A_4 case when the turbulence evolves at high compressibility. The results of Fig3 (a, b), fig4(a, b) and fig5(a, b) show that there is a decrease in the magnitude of the normalized production term $-2b_{12}$ when M_{g0} increase. The effect of compressibility of the other components is also interest that there is an increase in the transverse and stream wise anisotropies from case A1 to A4.

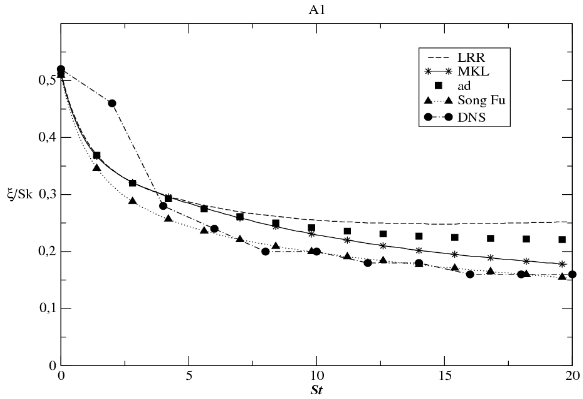

Fig 6.a. The time evolution of the normalized dissipation (ε/Sk) in case A1

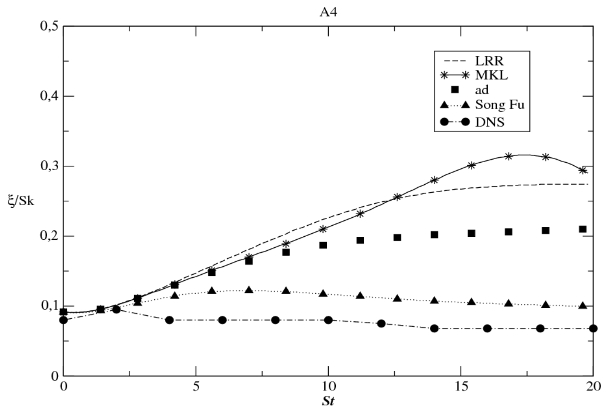

Fig 6.b. The time evolution of the normalized dissipation (ε/Sk) in case A4

Fig 6(a, b) shows that there is a decrease of (ε/Sk) when M_{g0} increases. It is clearly that the primary reason of the decrease in (ε/Sk) ($\varepsilon/Sk=-2b_{12}\ \varepsilon_s/P$) is the reduced level of the production term $-2b_{12}$.

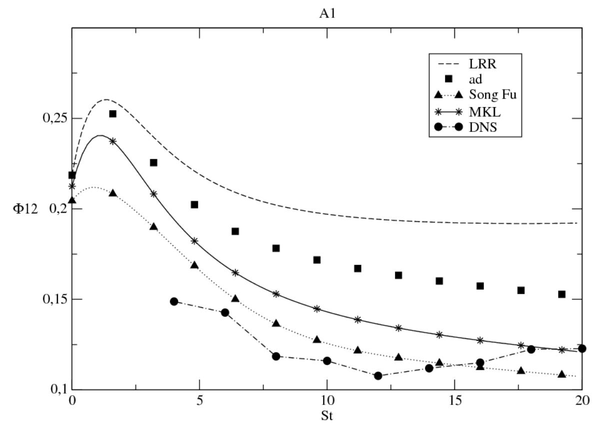

Fig 7.a. Time evolution of the pressure-strain correlation Φ_{12} in case A1

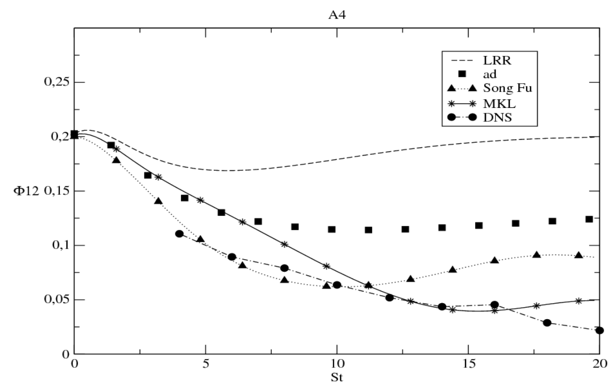

Fig 7.b. Time evolution of the pressure-strain correlation Φ_{12} in case A4

Fig 7(a, b) show the historical time of the components of the pressure-strain Φ_{12}, these results are in disagreement with the DNS results of Sarkar, this especially in the A4 simulation case where the turbulent compressible flow is highly.

Conclusion:

In this study, the widely used second order closure has been used for the prediction of compressible homogeneous turbulent shear flow.

The standard model for the pressure-strain correlation of L.R.R in conjunction with terms proposed by Sarkar yields poor predictions for compressible homogeneous shear flow. It was found that the dilatational terms are much smaller to reflect the correct physics of compressibility. An extension of the Launder, Reece and Rodi model has been proposed, the coefficients C1, C3 and C4 can be a function of the turbulent Mach number.

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