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Entropy Generation in Double Diffusive Convection with Soret Effect through a Square Porous Cavity

Nejib Hidouri[1], Ali Mchirgui[1*], Mourad Magherbi[2], Ammar Ben Brahim[1]

[1]*Gabès University, Engineers National School, Chemical and Process Engineering Department, Applied Thermodynamics Unit, Omar Ibn El Khattab Street, 6029 Gabès, Tunisia*

[2]*Gabès University, High Institute of Applied Sciences and Technology, Applied Thermodynamics Unit, Omar Ibn El Khattab Street, 6029 Gabès, Tunisia*

Abstract: This paper consists of a numerical investigation about the influence of Soret effect on entropy generation in double diffusive convection. It consists of a square porous Darcy – Brinkman cavity saturated by a binary perfect gas mixture and submitted to horizontal thermal and concentration gradients. The Control Volume Finite-Element Method was employed in order to solve the set of equations describing the studied phenomena. Influence of the Soret effect on the different irreversibilities was carried out by the variation of the thermal diffusion ratio and the Darcy number. Taking into consideration the Soret effect induces an increase of heat transfer and viscous irreversibilities. The diffusive entropy generation presents different behaviours.

Key words: Numerical method, Soret effect, entropy generation, porous medium, cavity, double diffusive convection.

1. Introduction

Double diffusive Convection through a fluid saturated porous medium is of fundamental interest in many engineering, industrial, and environmental applications such as seawater flow and mantle flow in the earth's crust, as well as in cooling of electronic devices and air-conditioning systems. There have been several reported studies dealing with heat and mass transfer in a porous medium. Kramer et al. [1] used the boundary domain integral method to study double diffusive natural convection in porous media. Haddad et al. [2] evaluated the dependence of velocity and temperature on the problem parameters. Valencia and Frederick [3] numerically studied the natural convection of air square cavities with Half-active and half-insulated vertical walls for various Rayleigh numbers. They showed that the heat transfer rates could be controlled to a certain extent, by varying the relative positions of the hot and cold elements. Double diffusive convection in a fluid-saturated porous medium was studied by Cheng [4, 5], near a permeable horizontal cylinder of an elliptic cross section with uniform wall temperature and concentration [4], and from an arbitrarily inclined plate in a fluid saturated porous medium with variable wall temperature and concentration [5].

The Ludwig-Soret effect occurs when a temperature gradient applied to a fluid mixture induces a net mass flow, which leads to the formation of a concentration gradient. Large number of papers has been published dealing with convection and entropy generation by taking into consideration the Soret effect. Hurle and Jakeman [6] theoretically analysed the Soret effect on

*** Corresponding author:** Ali Mchirgui
E-mail: ali.mchirgui@voila.fr

the Rayleigh–Jeffrey problem. They showed that stable solutions could occur owing to this effect, in water–methanol mixtures when they are heated from below. Gaikwad et al. [7] analytically analysed the double diffusive convection with Soret effect in a horizontal anisotropic porous layer. The influence of the couple stress parameter on the stability and on the heat and mass transfer was investigated. Analytical and numerical studies of natural convection with Soret effect in a shallow rectangular cavity filled with a micropolar fluid were carried out by Alloui and Vasseur. [8]. Tsai and Huang [9] numerically studied natural convection with cross effects of Soret and Dufour along a vertical surface with variable heat fluxes in a porous medium. It was found that the flow velocity and concentration distributions decrease with the decreasing of Soret number or the increasing of Dufour number. The analysis of entropy generation in double diffusive convection in a square cavity was performed by Hidouri et al. [10, 11]. It was concluded that Soret effect could not be neglected because it considerably affects the magnitude of entropy generation especially at higher Grashof numbers. Hidouri and Ben Brahim [12] numerically studied the influence of Soret and Dufour effects on entropy generation in transient double diffusive convection through a square cavity, filled with a binary perfect gas mixture.

The present paper reports a numerical investigation about the influence of Soret effect on entropy generation in double diffusive convection through a square porous cavity saturated by a binary perfect gas mixture.

2. Mathematical formulation

The considered system consists of a square cavity filled with a binary perfect gas mixture saturating a porous medium. The porous medium is assumed to be homogeneous, isotropic and in thermodynamic equilibrium with the fluid. Left and right walls are submitted to different but uniform temperatures and concentrations (T_h, C_h) and (T_c, C_c), respectively while the two horizontal walls are insulated and adiabatic. The density of the fluid satisfies the Boussinesq approximation and can be described as:

$$\rho(C,T) = \rho_0[1-\beta_T(T-T_0)-\beta_C(C-C_0)] \quad (1)$$

Using the Darcy-Brinkman model [13], the conservation equations of mass, momentum, energy and species diffusion are written in the dimensionless form as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (2)$$

$$\frac{1}{\varepsilon}.\frac{\partial u}{\partial \tau}+\frac{1}{\varepsilon^2}div(uU-\Lambda Pr\, grad\, u)=-\frac{Pr}{D_A}u-\frac{\partial p}{\partial x} \quad (3)$$

$$\frac{1}{\varepsilon}.\frac{\partial v}{\partial \tau}+\frac{1}{\varepsilon^2}div(vU-\Lambda Pr\, grad\, v)=-\frac{Pr}{D_A}v-\frac{\partial p}{\partial y}+G_{rT}(\theta+N\phi) \quad (4)$$

$$\sigma\frac{\partial \theta}{\partial \tau}+div(\theta U-R_k\, grad\,\theta)=0 \quad (5)$$

$$\varepsilon\frac{\partial \phi}{\partial \tau}+div(\phi U-\frac{\varepsilon}{Le}grad\phi)=\frac{\varepsilon}{Le}[div\,(K_T\, grad\theta)] \quad (6)$$

The governing equations are obtained by employing the following dimensionless variables:

$$u=\frac{u^*}{W};\quad v=\frac{v^*}{W};\quad x=\frac{x^*}{a};\quad y=\frac{y^*}{a};\quad \tau=\frac{t.W}{a};$$
$$p=\frac{P-P_0}{\rho_0 W^2};\quad \theta=\frac{T-T_0}{\Delta T};\quad \phi=\frac{C-C_0}{\Delta C}\quad \Delta T=T_h-T_c;$$
$$\Delta C=C_h-C_c;\quad N=\frac{G_{rS}}{G_{rT}};\quad G_{rT}=\frac{g\beta_T\,\Delta T a^3}{\upsilon^2};$$
$$G_{rS}=\frac{g\beta_C\,\Delta C a^3}{\upsilon^2};\quad Pr=\frac{\nu}{W.a};\quad \Lambda=\frac{\mu_{eff}}{\mu};$$
$$Da=\frac{K}{a^2};\quad \sigma=\frac{(\rho c)_m}{(\rho c)_f};\quad R_K=\frac{k_m}{k_f};\quad Le=\frac{\alpha_f}{D};$$
$$K_T=S\left(\frac{\beta_T}{\beta_S}\right)\left(\frac{\Delta T}{\Delta C}\right) \quad (7)$$

The thermal porous Rayleigh number will be used in the analysis: $Ra^* = PrDaGr_T$.

Heat and mass transfer rates between the heated wall and the medium are given by the average Nusselt and Sherwood numbers, respectively. They are given by:

$$Nu = \int_0^1 (-\frac{\partial \theta}{\partial x})dy \tag{8}$$

$$Sh = \int_0^1 (-\frac{\partial \phi}{\partial x})dy \tag{9}$$

The dimensionless initial and boundary conditions are:

At $\tau = 0:$ $u = v = 0, p = 0, \theta = 1 - x$ and $\phi = 1 - x$, for the entire cavity.

At $x = 0$: $\phi = \theta = 1$

At $x = 1$: $\phi = \theta = 0$

At $y = 0$ and $y = 1:$ $\frac{\partial \theta}{\partial y} = \frac{\partial \phi}{\partial y} = 0$ (10)

3. Entropy generation

In double diffusive convection, entropy generation results from heat transfer, fluid flow and species diffusion. The volumetric entropy generation is therefore the sum of irreversibilities due to thermal gradients, viscous dissipation and concentration gradients. In such problem, the Darcy dissipation term should not be neglected compared to the clear fluid term in the contribution of fluid friction irreversibility. Entropy generation due to fluid friction denotes both the wall and the fluid layer shear stress and the momentum exchange at the solid boundaries (Hooman et al. (2008)). Under the previous assumptions, the expression of the volumetric entropy generation in double diffusive convection through a porous medium for a single diffusing species, in 2D approximation is given by (Hidouri et al.[11, 12] and Hooman et al. [14, 15]):

$$s_{gen} = \left(\frac{k_m}{T_0^2} + \frac{RD_{CT}}{T_0}\right)\left[\left(\frac{\partial T}{\partial x^*}\right)^2 + \left(\frac{\partial T}{\partial y^*}\right)^2\right]$$

$$+ \frac{RD_e}{C_0}\left[\left(\frac{\partial C}{\partial x^*}\right)^2 + \left(\frac{\partial C}{\partial y^*}\right)^2\right]$$

$$+ \left(\frac{RD_e}{T_0} + \frac{RD_{CT}}{C_0}\right)\left[\left(\frac{\partial C}{\partial x^*}\right)\cdot\left(\frac{\partial T}{\partial x^*}\right) + \left(\frac{\partial C}{\partial y^*}\right)\cdot\left(\frac{\partial T}{\partial y^*}\right)\right]$$

$$+ \frac{\mu}{T_0 . K}(u^{*2} + v^{*2})$$

$$+ \frac{\mu}{T_0}\left[2\left(\frac{\partial u^*}{\partial x^*}\right)^2 + 2\left(\frac{\partial v^*}{\partial y^*}\right)^2 + \left(\frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*}\right)^2\right] \tag{11}$$

The first term of equation (11) is the entropy generation due to thermal gradients, the second is due diffusion and composed by a term due to pure concentration gradients and a sum of mixed products of temperature and concentration gradients. The third term denotes irreversibility due to viscous dissipation. The dimensionless form of local entropy generation is obtained by using the dimensionless variables previously listed, it takes the following form:

$$N = N_\theta + N_d + N_f \tag{12}$$

where:

$$N_\theta = \left[\left(\frac{\partial \theta}{\partial x}\right)^2 + \left(\frac{\partial \theta}{\partial y}\right)^2\right]$$

$$N_d = \varphi_1\left[\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2\right] + \varphi_2\left[\left(\frac{\partial \phi}{\partial x}\right)\cdot\left(\frac{\partial \theta}{\partial x}\right) + \left(\frac{\partial \phi}{\partial y}\right)\cdot\left(\frac{\partial \theta}{\partial y}\right)\right] \tag{13}$$

$$N_f = Br_T^*\left\{u^2 + v^2 + Da\left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right]\right\}$$

Br_T^* is defined as the modified Darcy-Brinkman number in the thermal diffusion process and given by:

$$Br_T^* = \frac{Br_T}{Da.\Omega} = \frac{\varphi_3}{Da} \tag{14}$$

Br_T is defined as the Darcy-Brinkman number in the thermal diffusion process. It is given by:

$$Br_T = \frac{\mu . W^2}{(k_m + RT_0 D_{CT})\Delta T} \tag{15}$$

φ_1 and φ_2 and φ_3 are the irreversibility distribution ratios. They are given by:

$$\varphi_1 = \left(\frac{RD_e}{k_m + RT_0 D_{CT}}\right)\left(\frac{\Omega'}{\Omega^2}\right)\Delta C \tag{16}$$

$$\varphi_2 = \left(\frac{RC_0 D_e + RT_0 D_{CT}}{k_m + RT_0 D_{CT}}\right)\left(\frac{\Omega'}{\Omega}\right) \tag{17}$$

$$\varphi_3 = \left(\frac{\mu T_0}{k_m + RT_0 D_{CT}}\right)\left(\frac{W}{(\Delta T)}\right)^2 \tag{18}$$

Ω and Ω' are the temperature and the concentration ratios, respectively. They are given by:

$$\Omega = \frac{\Delta T}{T_0},\ \Omega' = \frac{\Delta C}{C_0} \tag{19}$$

The total dimensionless entropy generation is obtained by numerical integration over the entire cavity volume A. It is given by:

$$S_T = \int_A N.dA. \tag{20}$$

4. Numerical method

The used numerical method for discretizing the system of equations is a modified version of the control volume finite element method of Saabas and Baliga [16] adapted to standard-staggered grids, in which pressure and velocity components are calculated and stored. The combined continuity, momentum, energy and concentration equations are solved by using The SIMPLER algorithm of Patankar [17] in conjunction with an alternating direction implicit (ADI) scheme for performing the time evolution. Local entropy generation is then calculated at any nodal point of the cavity. The total entropy generation for the entire cavity is obtained by numerical integration. The grids of sizes 31x31 and 41x41 and 51x51 nodal points are used in the numerical compilation according to the values of Rayleigh and Darcy numbers. The accuracy of the user's numerical code written in FORTRAN language was obtained by comparing average values of Nusselt and Sherwood numbers with those given by Kramer et al.[1] and Laureat et al. [13] (Table1). As it can be seen, the results are in good agreement with those given by the literature.

4. Results and discussion

The considered medium is a square porous cavity filled with a binary perfect gas mixture characterized by Pr = 0.71 and Le = 1.2. The study mainly concerns the influence of thermodiffusion effect (Soret effect) on different irreversibilities. The following values of irreversibility coefficients are considered: $\varphi_1 = 0.5$, $\varphi_2 = 10^{-2}$ and $\varphi_3 = 10^{-6}$.

Table 1: Average Nusselt and Sherwood numbers for *Le* = *10, D_A= 10^{-1} , Ra* = 100*

N	D_A	Présent study	Karmer et al.[1]	Laureat et al.[13]
0	10^{-2}	Nu = 1.72	Nu = 1.70	Nu = 1.70
0	10^{-4}	Nu = 2.84	Nu = 2.83	Nu = 2.84
2	10^{-1}	Nu = 1.45 Sh = 1.45	Nu = 1.43 Sh = 1.43	- -

The operating parameters are in the following ranges: $10^{-4} \le Da \le 10^{-2}$, $50 \le Ra^* \le 300$, $N = 2$ *and* $0 \le K_T \le 4$. The porous medium proprieties are kept constant, they are given by: $\varepsilon = 0.5, \Lambda = 1,\ \sigma = 1,\ R_k = 1.$

Figs.1a and 1b show the heat transfer irreversibility versus thermal diffusion ratio for $Da = 10^{-2}$ and 10^{-4} respectively, at a fixed buoyancy ratio $N = 2$ and for different thermal porous Rayleigh numbers.

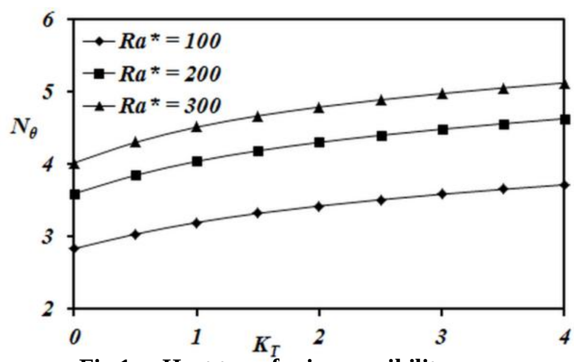

Fig.1 a: Heat transfer irreversibility versus thermal diffusion ratio (*Da* = *10^{-2}*, *N* = 2*)*

It can be seen that entropy generation due to heat transfer increases with the increase of thermal diffusion ratio. This is due to the increase of the average Nusselt number with the thermal diffusion ratio as illustrated in Fig.2. Thus, Soret effect induces the increase of heat transfer between the heated wall and the porous medium, inducing the increase of heat transfer irreversibility.

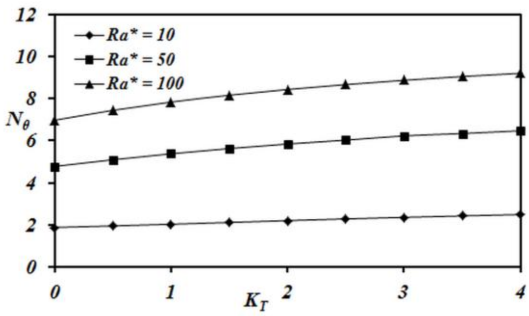

Fig.1 b: Heat transfer irreversibility versus thermal diffusion ratio (*Da* = 10^{-4}, *N* = 2)

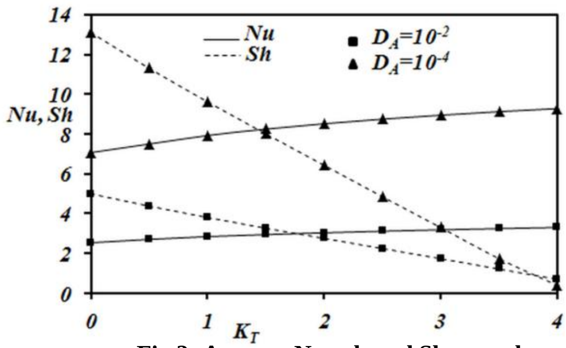

Fig.2: Average Nusselt and Sherwood numbers versus thermal diffusion ratio

Figs.3a and 3b show that the diffusive irreversibility decreases and reaches a minimum value for $K_T = 2$ then increases for $2 < K_T < 4$.

In fact, the entropy generation due to diffusion is a sum of two terms, the first is the irreversibility due to pure concentration gradient, and the second is a crossed term with both thermal and concentration gradients. Thus, the behaviour of diffusive irreversibility is driven by a competition between these tow terms.

The heat transfer expressed by Nusselt number increases with K_T (Fig.2) and the average mass transfer expressed by Sherwood number decreases in presence of Soret effect (Fig.2). The variation of diffusive entropy generation is diminutive for lower values of thermal diffusion ratio ($K_T \leq 2$) when the decrease of mass transfer predominates. For ($K_T > 2$), the augmentation of heat transfer due to Soret effect is more important which justify the behavior of entropy generation due to diffusion.

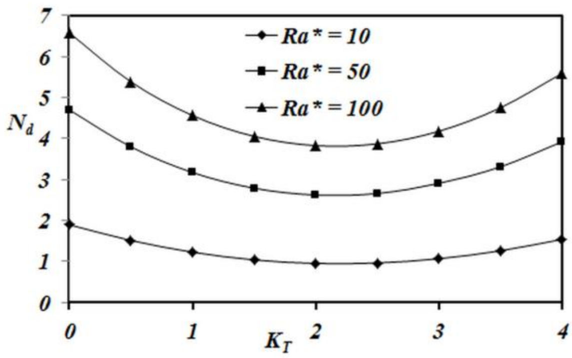

Fig.3 b: Diffusive irreversibility versus thermal diffusion ratio (*Da* = 10^{-4}, *N* = 2)

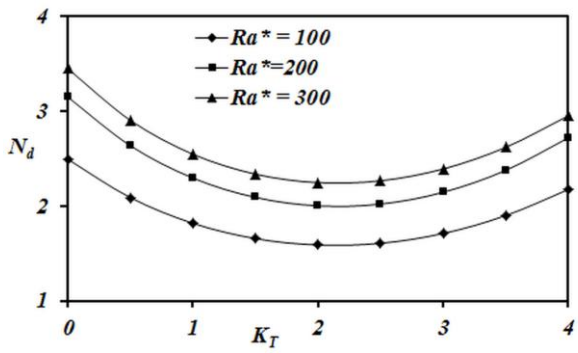

Fig.3 a: Diffusive irreversibility versus thermal diffusion ratio (*Da* = 10^{-2}, *N* = 2)

From Figs. 4a, it can be noticed that for $Da = 10^{-2}$, the fluid friction irreversibility can be neglected as compared to heat and mass transfer irreversibilities. Thus, entropy generation is mainly due to heat and mass transfer.

Irreversibility due to fluid friction is more important when decreasing the Darcy number (i.e. $Da = 10^{-4}$). It can be seen from Fig. 4b, that the increase of thermal diffusion ratio causes an augmentation of viscous irreversibility. This augmentation is more important for higher values of thermal porous Rayleigh number. This result is confirmed by Fig.5, showing the streamlines for the two studied cases of Darcy number.

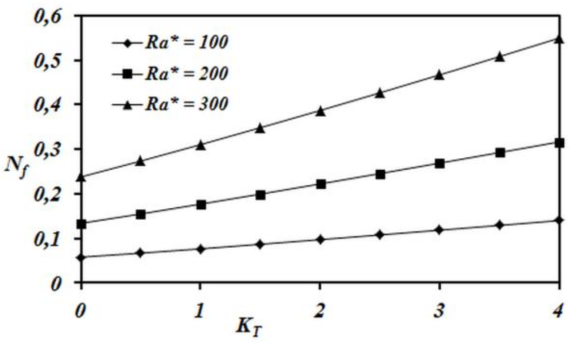

Fig. 4a: Fluid friction irreversibility versus thermal diffusion ratio (*Da* = 10^{-2}, *N* = 2)

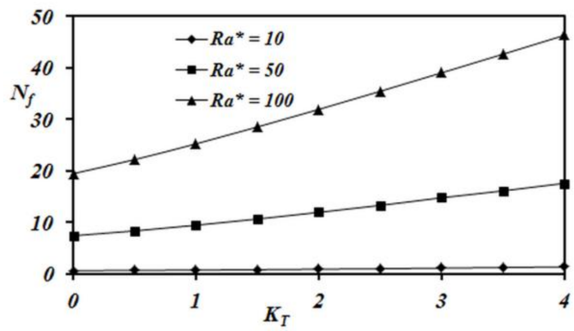

Fig.4b: Fluid friction irreversibility versus thermal diffusion ratio (*Da* = 10^{-4}, *N* = 2)

Fig.5 shows that the convective motion is accelerated by the decrease of medium permeability (i.e. the decrease of Darcy number) and by the increase of thermal diffusion ratio (i.e. presence of Soret effect).

4. Conclusion

A numerical study of entropy generation in double diffusive convection with Soret effect through a porous medium was investigated. The increase of thermal diffusion ratio causes the augmentation of heat transfer and fluid friction irreversibilities. It was found that entropy generation due to heat transfer and fluid friction increase with the decrease of Darcy number and with the increase of thermal diffusion ratio. The diffusive irreversibility takes a minimum value at $K_T = 2$ for the case of cooperative buoyancy forces.

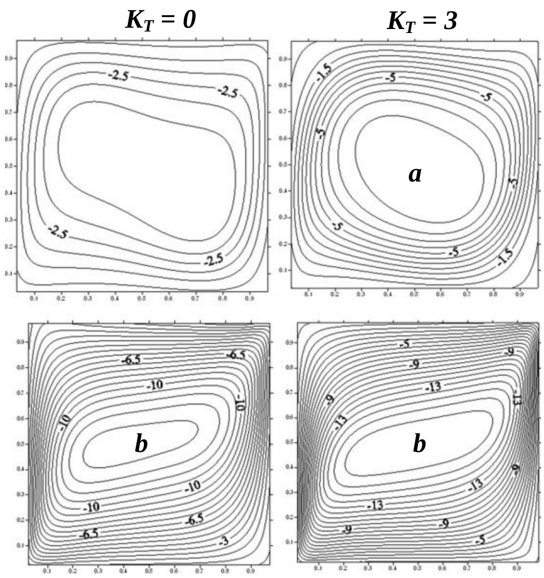

Fig.5: Streamlines (a: $D_A = 10^{-2}$/ b: $D_A = 10^{-4}$)

Nomenclature:

a: characteristic length (m)

C: concentration (mol· m^{-3}).

Da: Darcy number.

D_{CT}: thermal diffusion coefficient in the porous medium ($mol.m^{-1}.s^{-1}.K^{-1}$)

D_e: molecular diffusivity through porous matrix, ($m^2.s^{-1}$).

g: gravitational acceleration ($m·s^{-2}$).

G_{rT}: thermal Grashof number.

G_{rS}: solutal Grashof number.

k: thermal conductivity ($W.m^{-1}.K^{-1}$).

K: permeability of the porous medium (m^2).

K_T: thermal diffusion ratio.

Le: Lewis number.

N: buoyancy ratio (G_{rS}/ G_{rT})

P: pressure ($kg·m^{-1}.s^{-2}$).

p: dimensionless pressure.

Pr : Prandtl number.

Ra : Rayleigh number.

R_K: thermal conductivity ratio (k_m/k_f).
Ra^*: porous thermal Rayleigh number.
S: Soret parameter
S_T: dimensionless total entropy generation.
Sc : Schmidt number.
Sh: Sherwood number.
T: temperature (K).
t : time (s)
u, v : dimensionless velocity components.
u^*, v^*: velocity components along x^*, y^* directions ($m \cdot s^{-1}$).
x, y : dimensionless Coordinates.
x^*, y^*: Cartesian coordinates (m).
W : characteristic Vvelocity ($m \cdot s^{-1}$).
Greek symbols:
ν : kinematic viscosity ($m^2.s^{-1}$).
β_T : thermal volumetric expansion coefficients (K^{-1}).
β_C : solutal volumetric expansion coefficients ($m^3.mol^{-1}$).
Λ : viscosity ratio (μ_{eff} / μ)
ε : porosity of the medium.
μ : fluid dynamic viscosity ($kg.m^{-1}.s^{-1}$).
μ_{eff} : viscosity in the Brinkman model ($kg.m^{-1}.s^{-1}$).
ϕ : dimensionless concentration.
ρ : fluid density ($kg.m^{-3}$).
σ : specific heat ratio [$(\rho c)_m / (\rho c)_f$].
θ : dimensionless temperature.
τ : dimensionless time.

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