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A theoretical study of a thermal radiation inverse problem for the estimation of the optical thickness by means of an optimization technique

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Abstract. An inverse radiation problem was considered to estimate the optical thickness for a one-dimensional cylindrical model of a semi-transparent, gray and isotropically scattering medium. The radiative transfer equation is solved using the finite volume method and the temperature is determined according to the dimension of the semi-transparent medium. In order to find the points which give us more information about the optical thickness using the measured temperatures, we carried out a sensitivity analysis. The solution of the inverse problem is obtained with the Levenberg-Marquardt method. The identification results were analyzed with respect to the number of measurements and the initial estimate of the unknown radiative property. The effect of the parameter of the LM method on the stability of the solution, in particular in the vicinity of the initial estimate, was also investigated.

Key words: inverse problem, Levenberg-Marquardt, optical thickness, sensitivity analysis

1. Introduction

Heat transfer in semi-transparent media heated to high temperatures is a very topical field of investigation due, on the one hand to the high industrial interest of such a study (notably for the boilers, the furnaces and the combustion chambers where radiation heat transfer is dominant) and on the other hand, to the recent progresses observed in the numerical processing possibilities for the equation of radiative transfer in multidimensional geometries. The determination of the parameters which characterize the radiative transfer, such as the optical thickness and the scattering albedo is very important to control this phenomenon.

Inverse radiation analysis has been concerned with the estimation of radiative properties from measured radiation quantities [1,2]. The determination of medium properties, such as the extinction coefficient, the absorption coefficient, the scattering albedo, the phase function, the optical thickness, and a gas temperature, as well as surface properties, such as the emissivity and a boundary temperature has been achieved by inverse radiation analysis from measured intensities or temperatures [3–8].

The aim of the present work is to estimate, through the solution of an inverse problem of thermal radiation, the optical thickness for a one-dimensional cylindrical model of a semi-transparent, gray and isotropic scattering medium.

The solution of the inverse problem is obtained with the Levenberg-Marquardt method.

The inverse analysis using the Levenberg-Marquardt method consists in minimizing an error function representing the difference between predictions and experimental measurements of the responses of the studied system. In heat transfer, this method was used by Sawaf et al. [9] in the simultaneous estimation of the thermal conductivity and the volumetric heat capacity (functions of temperature), Lazard et al. [10] in determining the diffusivity of a semi-transparent medium and Kanevce et al. [11] in the identification of a diffusion coefficient (depending on the temperature). Mejias et al. [12] used two versions of the Levenberg-Marquardt method and four versions of the conjugate gradient method for comparing the identification results of the thermal conductivity values in three directions.

The radiative transfer equation is solved using the finite volume method and the temperature is determined according to the dimension of the semi-transparent medium. A sensitivity analysis is carried out in order to find the points which give us more information about the optical thickness using the measured temperatures.

Identification of the optical thickness is carried out using simulated measurements of temperature where exact and noisy values are considered. Effects of the number of measurements and the noise level on the inverse solution are studied. The convergence of the

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LM method is analyzed when using initial estimates sufficiently distant from the exact values. The effect of the parameter of the LM method on the stability of the solution, in particular in the vicinity of the initial estimate, is also investigated.

2. Formulation of the direct problem

2.1. Model description

As shown in Figure 1, the model under study is a semi-transparent medium confined between two infinite coaxial cylinders of radius r_e and r_i (r_e>r_i) whose surfaces are maintained respectively to the temperatures T_e and T_i (T_i>T_e). The medium is considered gray and of isotropically scattering and the boundary surfaces are assumed to be diffusely emitting and reflecting, having the same emissivity.

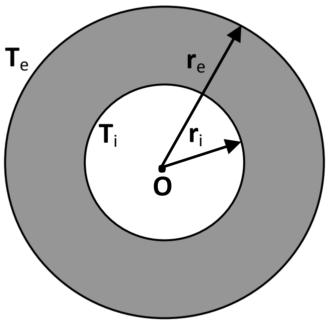


Fig. 1. Schematic of the physical system

2.2. Radiative transfer equation (RTE)

The radiative transfer equation for absorbing-emitting and isotropically scattering medium, is writen as

$$\frac{\partial I(s,\vec{\Omega})}{\partial s} + \beta I(s,\vec{\Omega}) = \beta R \qquad (1)$$

where

$$R = (1-\omega)I^0(s) + \frac{\omega}{4\pi}\int_{4\pi} I(s,\vec{\Omega}')d\Omega' \qquad (2)$$

I(s,$\vec{\Omega}$) is the radiation intensity in the direction $\vec{\Omega}$ at the position s. I^0 is the blackbody radiation intensity.

The radiative boundary condition for a diffusely emitting and reflecting wall, is written as

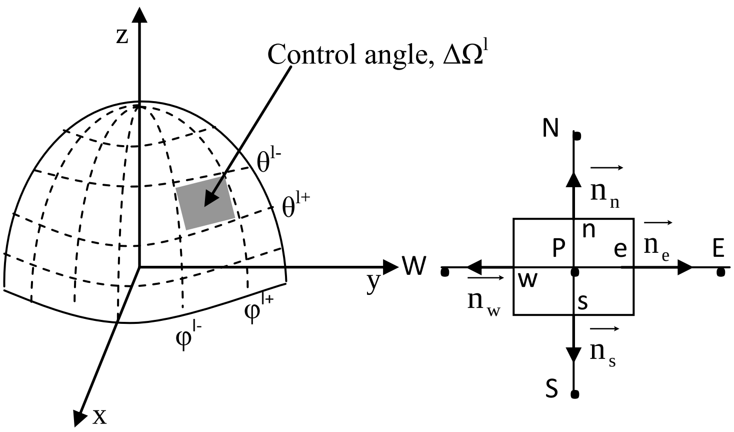
$$I(s,\vec{\Omega}) = \varepsilon I^0(s) + \frac{1-\varepsilon}{\pi}\int_{\vec{\Omega}'\vec{n}<0} I(s,\vec{\Omega}')\left|\vec{\Omega}'\vec{n}\right|d\Omega' \qquad (3)$$

where $\vec{n}$ is a unit vector normal to the control volume surface.

2.3. The finite volume method

The finite volume method for radiative heat transfer divides the computational domain into a finite number of control volumes and the total solid angle into an arbitrary number of solid angles (Figure2). The control solid angle $\Delta\Omega^l$ was calculated analytically by

$$\Delta\Omega^l = \int_{\theta^{l-}}^{\theta^{l+}}\int_{\varphi^{l-}}^{\varphi^{l+}} \sin\theta d\theta d\varphi \qquad (4)$$



(a) (b)

Fig. 2. (a) Control solid angle and (b) Spatial control volume.

The integration of equation (1) over an arbitrary control volume Δv and a control angle $\Delta\Omega^l$ gives

$$\int_{\Delta\Omega^l}\int_{\Delta A} I\vec{\Omega}\vec{n}dAd\Omega = \int_{\Delta\Omega^l}\int_{\Delta v} \beta(R-I)dvd\Omega \qquad (5)$$

By assuming constant the magnitude of the intensity but allowing its direction to vary within the control volume and the control angle, the following finite-volume formulation can be obtained

$$\sum_{i=1}^{4} I_i^l \Delta A_i N_i^l = \beta(R - I^l)\Delta v \qquad (6)$$

with

$$R = (1-\omega)I^0 + \frac{\omega}{4\pi}\sum_{l'=1}^{L} I^{l'}\Delta\Omega^{l'} \qquad (7)$$

The term N_i^l, evaluated analytically, takes into account the variation of the intensity direction within $\Delta\Omega^l$,

$$N_i^l = \frac{1}{\Delta\Omega^l}\int_{\Delta\Omega^l} \vec{\Omega}\vec{n_i}\, d\Omega \qquad (8)$$

Equation (6) indicates that a net outgoing radiant energy across the control-volume faces must be balanced by a net generation of radiant energy within the control volume and the control angle.
Using the step scheme (Chai et al [13]), equation (1) becomes

$$a_p^l I_p^l = a_W^l I_W^l + a_E^l I_E^l + a_S^l I_S^l + a_N^l I_N^l + b_p \quad (9)$$

where

$$a_W^l = \Delta A_w \max[-N_w^l, 0] \qquad (10)$$

$$a_E^l = \Delta A_e \max[-N_e^l, 0] \qquad (11)$$

$$a_N^l = \Delta A_n \max[-N_n^l, 0] \qquad (12)$$

$$a_S^l = \Delta A_s \max[-N_s^l, 0] \qquad (13)$$

$$a_p^l = \Delta A_w \max[N_w^l, 0] + \Delta A_e \max[N_e^l, 0] + \Delta A_s \max[N_s^l, 0] + \Delta A_n \max[N_n^l, 0] + \beta\Delta v_p \qquad (14)$$

$$b_p = \beta R_p . \Delta v_p \qquad (15)$$

The radiative boundary condition (equation (3)) for a diffusely emitting and reflecting wall can be discretized as

$$I_F^l = \frac{\varepsilon\sigma T_F^4}{\pi} + \frac{(1-\varepsilon)}{\pi}\sum_{L+}\left|N_F^{l'}\right| . I_F^{l'} . \Delta\Omega^{l'} \quad (16)$$

3. The Levenberg-Marquardt method

The inverse problem studied in this paper is regarded as an optimization problem of parameter estimation where the Levenberg-Marquardt method is applied as the estimation technique. This method is an iterative procedure based on the minimization of a cost function and whose algorithm is a combination of the Gauss method and the Steepest Descent method. The solution of the problem is obtained when the vector of unknown parameters **P** minimizes the following sum of squares function [14]:

$$S(P) = \sum_{i=1}^{M} (Y_i - T_i(P))^2 \qquad (17)$$

where M is the number of sensors, **Y** is the vector of measured temperatures and **T**(**P**) is the vector of calculated temperatures obtained from the solution of the direct problem by using the current available estimate for the unknown parameters vector **P**. The norm of squared residues S is minimized by differentiating equation (18) with respect to each of the unknown parameters P_j (j =1…p) and then setting the resulting expression equal to zero yielding to the following set of algebraic equations:

$$\sum_{i=1}^{M} \frac{\partial T_i}{\partial P_j}(Y_i - T_i(P)) = 0 \qquad (18)$$

Due to the nonlinearity of this system of equations, an iterative technique is necessary for its solution. The iterative procedure of the Levenberg-Marquardt method is given by [14,16]:

$$P^{n+1} = P^n + \left[(J^n)^T (J^n) + \mu^n I\right]^{-1} (J^n)^T \left[Y - T(P)\right] \qquad (19)$$

where **J** is the jacobian matrix which elements, known as the sensitivity coefficients, are written as:

$$J_{ij} = \frac{\partial T_i}{\partial p_j} \quad (i = 1,2,\ldots\ldots, M \text{ and } j = 1,2,\ldots\ldots, p) \quad (20)$$

and μ *is* a damping parameter added to the diagonal of ($\mathbf{J}^T\mathbf{J}$) in order to control the stability of the algorithm. Iterations are generally started with large values of μ. Then this parameter is gradually reduced as the solution approaches the converged result. In this one unknown parameter case, the vector **P** is reduced to a scalar quantity (**P**=τ) (optical thickness) and the jacobian matrix is reduced to a vector written as:

$$J = \left[\frac{\partial T_1}{\partial \tau}, \frac{\partial T_2}{\partial \tau} \ldots\ldots \frac{\partial T_M}{\partial \tau}\right] \quad (21)$$

4. Results and discussion

4.1. Sensitivity analysis

The solution of the direct problem is obtained by considering known all the radiative parameters. Indeed the RTE is solved using the finite volume method, which allowed us to determine the temperature distribution in the participating medium confined between the two concentric cylinders. The spatial domain is divided into Nr = 20 control volumes. The solid angle is divided into (2 × 12) elementary solid angles. We put:

$$T^* = \frac{T - T_e}{T_i - T_e} \text{ and } r^* = \frac{r - r_i}{r_e - r_i} \qquad (22)$$

Our model is a semi-transparent medium confined between two coaxial infinitely long cylinders, where the two cylindrical surfaces are respectively brought to the temperatures ($T_i^* = 1$ and $T_e^* = 0$). Figure 3 shows the variation of the temperature (T*) depending on the radius (r*) for different values of the optical thickness. It is noted that increasing the optical thickness increases the temperature level. This is because if the medium becomes optically thick, radiative energy will be absorbed by the medium which sees its temperature increases. In the opposite case, the radiative energy is easily transferred to the cold surfaces where the temperature of the medium decreases.

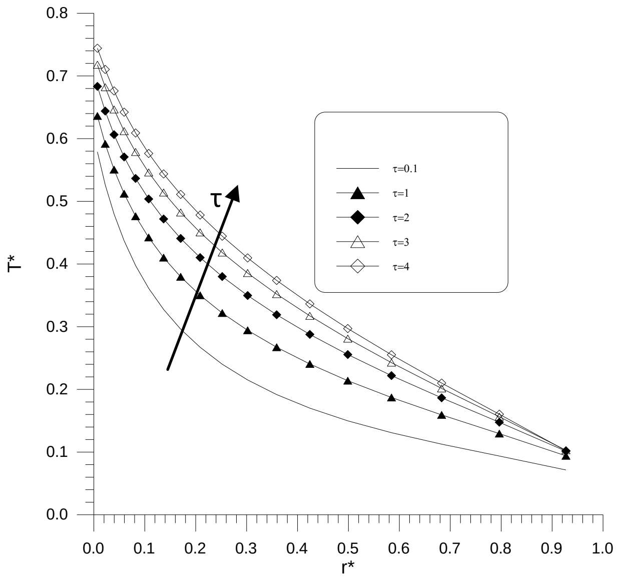


Fig.3. Influence of the optical thickness on the temperature

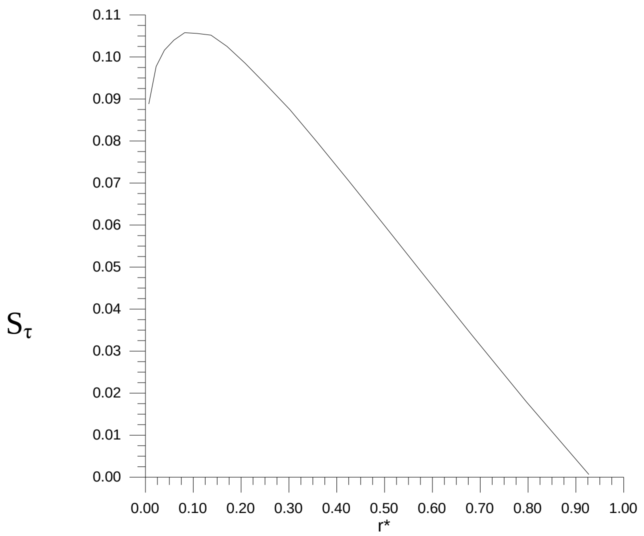


Fig.4. Reduced sensitivity to the optical thickness

Before starting the identification, a sensitivity analysis should be carried out in order to choose the points of temperature measurements that give us more information on the optical thickness. We define the reduced sensitivity to the optical thickness by the following expression:

$$S_\tau(r^*) = \tau \frac{\partial T^*}{\partial \tau} \qquad (23)$$

Figure 4 shows the variation of the reduced sensitivity to the optical thickness as a function of r*. We note that the sensitivity is greater away from the end (r * = 1) and approaching the end (r * = 0). Thus, it is wiser to place the points of temperature measurement for the identification of the optical thickness nearest to the inner cylinder.

4.2. Identification

In this section, we focus on the identification of a single radiative property (optical thickness) based on the results of the direct problem and the sensitivity analysis, and we study the effect of some parameters on the identification.

For the measurements needed in the identification, we use noisy simulated temperatures obtained by adding a random error (white, additive, uncorrelated, with zero mean and constant variance) to the solution of the direct problem, as follows [15]:

$$Y = T_{exact} + \omega_r . \sigma \qquad (24)$$

where σ is the standard deviation of measurement errors and ω_r is a random number lying, with 99% of probability, in the range $-2.576 < \omega_r < +2.576$ if normal (Gaussian) distribution of errors is considered.

T_{exact} is the temperature calculated using the solution of the RTE using the exact value of the optical thickness (τ_{exact}=4).

4.2.1. Identification with exact temperatures

Figure 5 represents the identification of the optical thickness using exact temperatures. For the three curves 5 (a, b and c) we obtain the convergence of the Levenberg-Marquardt method to the exact value of the optical thickness.

Figure (5.a), shows the effect of the Levenberg-Marquardt parameter μ° on the identification. The iterative algorithm reaches the exact value of the optical thickness faster by reducing μ° from 100 to 0,001. It is noted from Figure (5.b) that we can reach the same exact value by choosing an initial value of the optical thickness sufficiently far from the exact value (τ° = 0.01 and τ° =7).

Figure (5.c) represents the identification of the optical thickness by varying the number of measurement points (n_p). By increasing n_p we notice that the identification becomes better, which is explained by the fact that we have more information about the optical thickness.

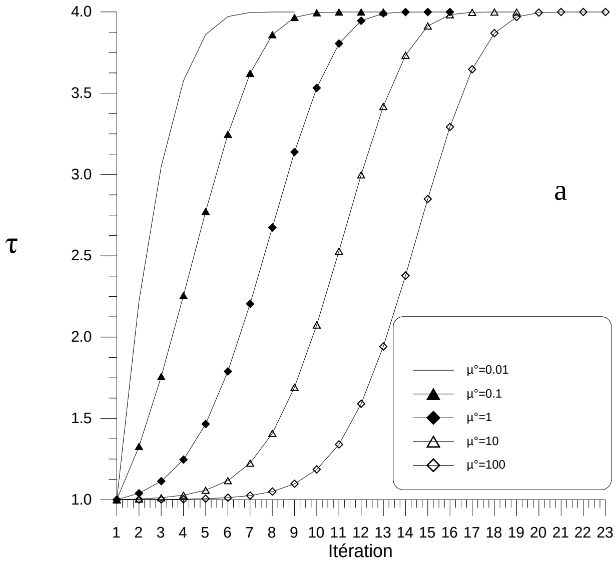


Fig.5 (a): Identification of the optical thickness with τ^0=1 and n_p=15, for different values of µ°.

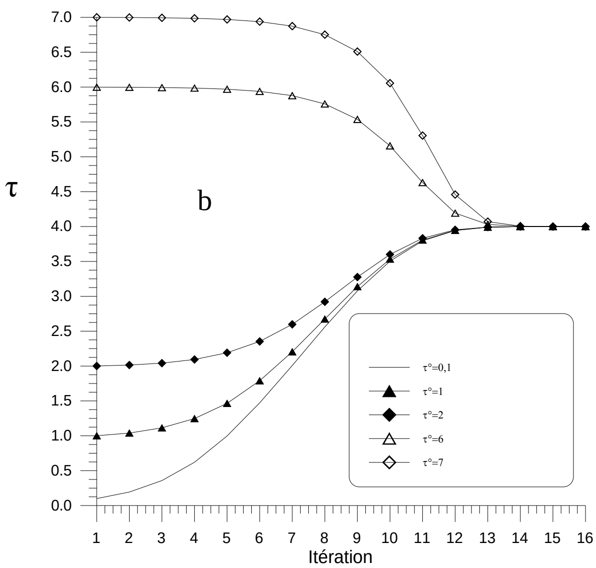


Fig.5 (b): Identification of the optical thickness with µ° = 1 and n_p = 15, for different values of the initial choice of the optical thickness (τ°)

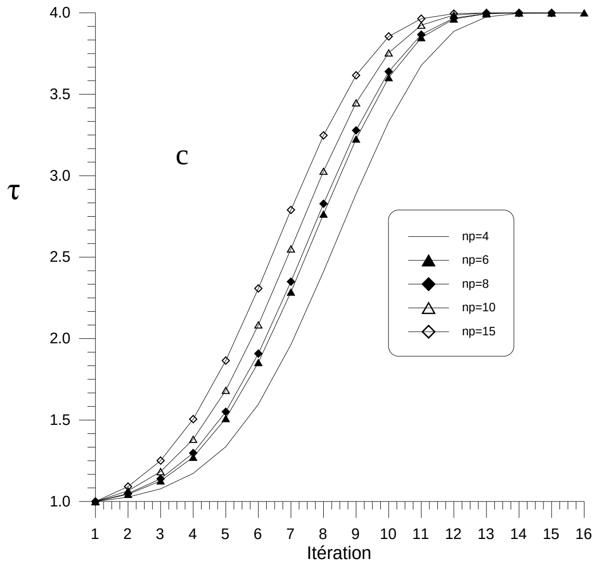


Fig.5 (c): Identification of the optical thickness with µ° = 1 and τ° = 1, for different number of measurement points (n_p) .

4.2.2. Identification with noisy simulated temperatures

Figure 6 represents the identification of the optical thickness during the iterations for different values of the standard deviation of the measurement noise σ. It is clear that the identification is better for lower values of σ (Table1).

Table 1. Effect of the noise level of the measurements on the precision of the identification

σ (K)	0	0.1	0.5	1	5
τ	3.9999	3.9603	3.8056	3.6215	2.4393
Error (%)	10^{-4}	1	4.86	9.46	39

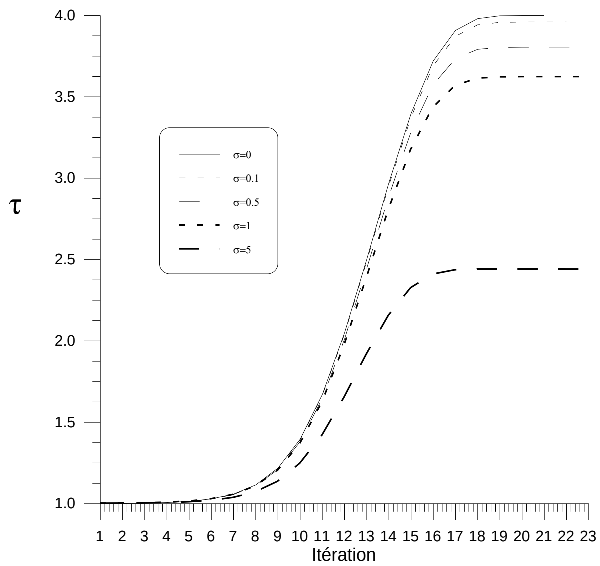


Fig.6. Identification of the optical thickness (τ) with noisy simulated temperatures

5. Conclusion

The finite volume method was used to solve an inverse problem of radiative transfer in a semi-transparent gray absorbing-emitting and isotropically scattering medium represented by a one-dimensional cylindrical geometry, with the aim of identifying the optical thickness. The radiative transfer equation, solved using the finite volume method, allowed us to determine the temperature distribution in the semi-transparent medium. To investigate the feasibility of identifying the optical thickness, we performed a sensitivity analysis, which allowed us to locate the points containing best information on the radiative property to identify. Indeed, the sensitivity is higher, approaching the inner boundary of the semi-transparent medium. Thus, it is more useful for better identification of the optical thickness, selecting measurement points closer to the inner cylinder. The solution of the radiative inverse problem was performed using the Levenberg-Marquardt method whose convergence depends mainly on the Levenberg-Marquardt parameter $\mu°$, the initial estimate $\tau°$ and the number of measurement points n_p.

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