

Effect of Rayleigh number on the turbulent structures with boundary layer in a differentially heated

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Abstract: The heat transfer by convection is so far a basic principle in many industrial applications. This study led to the analysis of turbulent convection. $Ra > 10^9$ in a three-dimensional parallelepiped cavity filled with air, the two opposite vertical walls are differentially heated at a constant temperature, the other walls are hot wall except ceiling. The finite volume method has been used to discretized the equations of flow in turbulent convection turbulence model used is $(\kappa-\epsilon)$.

The results are relevant because they show that for a number of $Pr = 0.71$ the Rayleigh number, thus generating a great influence on heat transfer within the study area, and the onset of instability due to the interaction of turbulent structures with the boundary layer.

Key words: Natural convection, finite volume, parallelepiped.

1. Introduction

The case of the rectangular cavity with differentially heated vertical walls is a basic configuration of various industrial devices, and especially a reference case very simple for the development and validation of the numerical simulation of convection natural. The study of natural convection of fluids in the cavities has been a large number of both theoretical and experimental work. The interest of this study lies in its involvement in many natural and industrial, such as cooling of electronic circuits and nuclear reactors, building insulation (case of double glazing), metallurgy, crystal growth for the semiconductor industry, etc. The fluid flow, whether laminar or turbulent regime are described by the system of partial differential equations. Thus, all physical phenomena are governed by the system formed by the equations of continuity, momentum and energy that must be resolved to know proceeding. The characteristics of the temperature field and flow field.

Unfortunately, it is virtually impossible to find an analytical solution and accurate systems such that the equations mentioned above are very complex, that is to say, non-linear and coupled on the one hand to the other. In this case, the use of numerical resolution is needed, and encourages us to choose the appropriate numerical method to obtain the best approximations.

1.1 Description of the problem

The physical model considered is shown schematically in Figure. 1. It is a three-dimensional rectangular cavity large ($H = 2.46$ m in height, $L = 0.385$ m width, $D = 0.72$ m deep) filled with air, two opposite vertical walls are differentially heated constant temperature, the two horizontal walls are differentially heated, the two vertical side walls are hot. We study this configuration. The flow in the cavity is turbulent $Ra > 10^9$.

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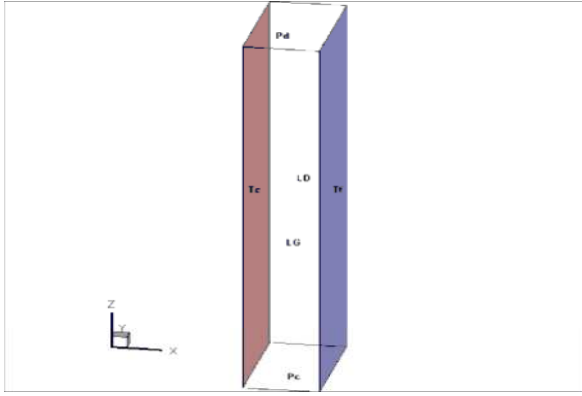


Fig. 1 Schematic of the cavity

T_c : hot wall

T_f : cold wall

LD : right side wall

LG : left sidewall

Pd : ceiling

Pc : Floor

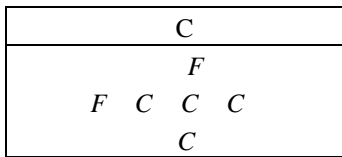
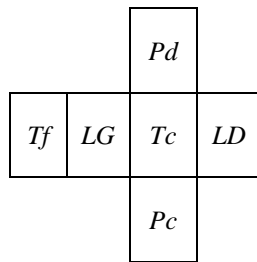


Fig. 2 configuration studied and their mode of representation

1.2 Boundary conditions

$$x = 0, y = 0, y = D, z = 0 \Rightarrow u = v = w = 0, T = T_c$$

$$x = L, z = H \Rightarrow u = v = z = 0, T = T_f$$

2. Numerical method

2.1 General transport equation

The general equation of a variable transmission operating in a three-dimensional flow incompressible, is written in the Cartesian system as follows:

$$\frac{\partial \Phi}{\partial \tau} + \frac{\partial}{\partial x_j} (U_j \Phi) = \frac{\partial}{\partial x_j} \left(\Gamma \frac{\partial \Phi}{\partial x_j} \right) + S_\Phi$$

With $j=1,2,3$ (summation index in the three-dimensional case)

$\frac{\partial \Phi}{\partial \tau}$: Term time transient

$\frac{\partial}{\partial x_j} (U_j \Phi)$: Term convective (transport by diffusion)

$\frac{\partial}{\partial x_j} \left(\Gamma \frac{\partial \Phi}{\partial x_j} \right)$: Term diffusive (diffusive transport)

S_Φ : source Term

with :

Table 1 Coefficients of the equations governing the phenomenon

equation	Φ	Γ	S_Φ
Continuity	I	0	0
Quantity move (x)	U	μ	$-\frac{\partial}{\partial x} \left(P + \frac{2}{3} \rho k \right) + \frac{\partial}{\partial x} \left((\mu_t + \mu) \frac{\partial U}{\partial x} \right) + \frac{\partial}{\partial y} \left((\mu_t + \mu) \frac{\partial V}{\partial x} \right) + \frac{\partial}{\partial z} \left((\mu_t + \mu) \frac{\partial W}{\partial x} \right)$
Quantity move (y)	V	μ	$-\frac{\partial}{\partial y} \left(P + \frac{2}{3} \rho k \right) + \frac{\partial}{\partial x} \left((\mu_t + \mu) \frac{\partial U}{\partial y} \right) + \frac{\partial}{\partial y} \left((\mu_t + \mu) \frac{\partial V}{\partial y} \right) + \frac{\partial}{\partial z} \left((\mu_t + \mu) \frac{\partial W}{\partial y} \right)$
Quantity move (z)	W	μ	$-\frac{\partial}{\partial z} \left(P + \frac{2}{3} \rho k \right) + \frac{\partial}{\partial x} \left((\mu_t + \mu) \frac{\partial U}{\partial z} \right) + \frac{\partial}{\partial y} \left((\mu_t + \mu) \frac{\partial V}{\partial z} \right) + \frac{\partial}{\partial z} \left((\mu_t + \mu) \frac{\partial W}{\partial z} \right)$
Turbulent kinetic Energy K	K	$\mu + \frac{\mu_t}{\sigma_t}$	$P K + G - \rho \varepsilon$
Dissipation ε	ε	$\mu + \frac{\mu_t}{\sigma_\varepsilon}$	$C_{\varepsilon 1 c} \frac{\varepsilon}{k} (P k + G) - C_{\varepsilon 2} \rho \frac{\varepsilon}{k}$
Energy	θ	K	0

with:

$$Pk = (\mu_t + \mu) \left[2 \left\{ \left(\frac{\partial U}{\partial x} \right)^2 + \left(\frac{\partial V}{\partial y} \right)^2 + \left(\frac{\partial W}{\partial z} \right)^2 \right\} + \left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} \right)^2 \right]$$

$$G = -\frac{\mu_t}{Pr} \frac{1}{\rho^2} \frac{\partial \bar{\rho}}{\partial x_i} \frac{\partial \bar{P}}{\partial x_i}$$

To obtain the equation discretization of the dependent variable in a volume Φ Cartesian equation is integrated on a general transport volume control.

2.2. Numerical model used by the Fluent code

Fluent code used finite volume method to discretize the equations transport. In this method one integrates the conservation equations that is applied to each elementary volume control. From the known variables at the centers of volumes, we evaluate the flux surfaces in volumes interpolation. The grid is a priori any, allowing to process flows with complex geometry. This method gives very good results, because it guarantees the conservation of mass and energy balances throughout the study area.

2.3 The numerical scheme

We will present a general form of the discretized algebraic equation where the total flow of convection and diffusion is calculated by a function, and after writing the discretized equation in its compact form we use in this calculation:

- The power-law scheme this scheme is better positioned to capture the physical phenomena of heat transfer.
- The discretization scheme of the pressure coupling speed is SIMPLE.
- The convergence criterion used is the under-relaxation.

2.4 Mesh

One of the questions that are asked is what type of mesh (regular or not tightened to the walls) used for different Rayleigh numbers and different configurations considered. This, as we consider the

results as satisfactory as soon as further refine the space does not significantly influence more. We initially tried to use a mesh with no regular space, but the results were not very satisfactory. In addition, the knowledge of boundary layer phenomena, it became clear that it was preferable to use a finer mesh to the walls. This type of mesh has been used for the remainder of the study. Is given in the following figure is an example of the tight mesh walls was created.

2.5. validation

In order to verify the accuracy of the numerical results obtained in the present work, a validation of the numerical code was made taking into account some numerical studies available in the literature. Ampofo results [12] obtained in the case of a square cavity containing air, were used to test our simulation by Fluent.

The comparison was made by considering the Rayleigh number 1.58×10^9 . Comparison of velocity profiles along the plane V medium shows a excellent agreement.

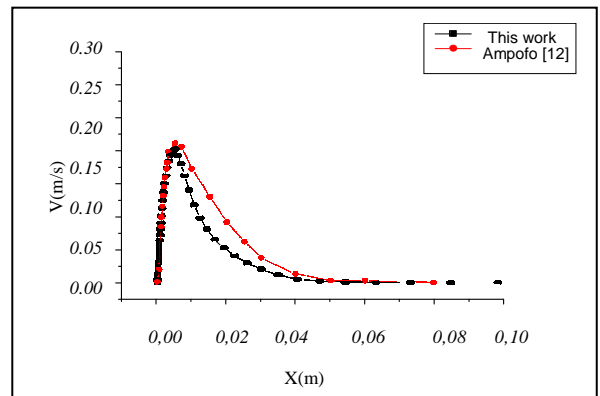


Fig. 3 Comparison of velocity profile V along the y = 0.375m.

3. Results: Analysis and Discussion

The effect of the increase of the Rayleigh number is increased convection. In what follows, we will fix the Prandtl number $Pr = 0.71$, varying the Rayleigh number based on ΔT .

$$\Delta T = 2^\circ C, Ra = 2,5.10^9.$$

$$\Delta T = 5^\circ C, Ra = 6,8.10^9.$$

$$\Delta T = 20^\circ C, Ra = 2,5.10^{10}.$$

$$\Delta T = 50^\circ C, Ra = 6,8.10^{10}.$$

$$\Delta T = 80^\circ C, Ra = 10^{11}.$$

The object is to capture the effects of this variation on the forces of buoyancy (Buoyancy). Fig . 4 represents the thermal field. In the fig.5

we represent the velocity vector field, we can see that the increase of the Rayleigh number generates recirculation zones. For the fig .6 we represent the vertical velocity field. We note that the maximum value augment with increasing Rayleigh number, the same remark applies to the horizontal velocity field represented in the fig . 7.

Regarding the vertical velocity profiles fig. 8, they show for different Rayleigh number, we can see that the maximum value of the vertical velocity augment with increasing Rayleigh number. In the fig.8 increasing the vertical velocity is proportional to the increase of the Rayleigh number is to say the increase in ΔT . Therefore the turbulence is directly related to a critical Rayleigh number must be set.

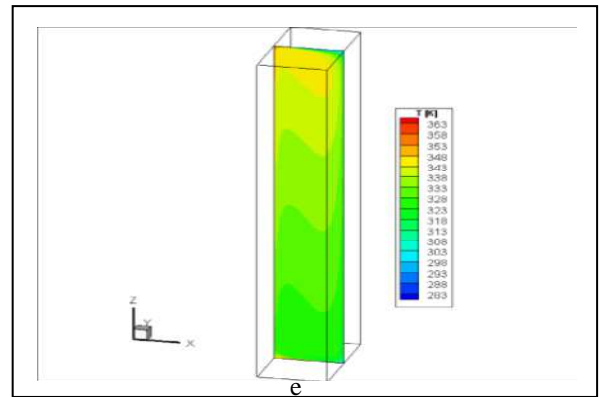
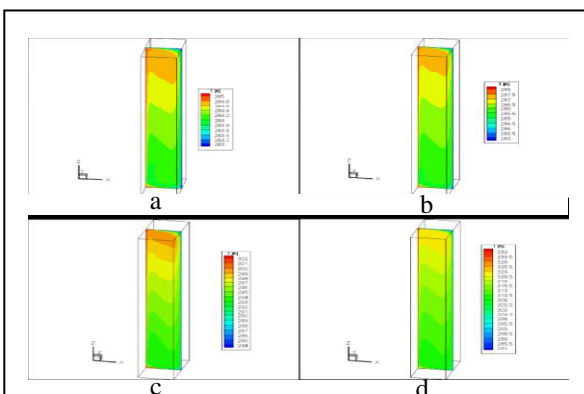
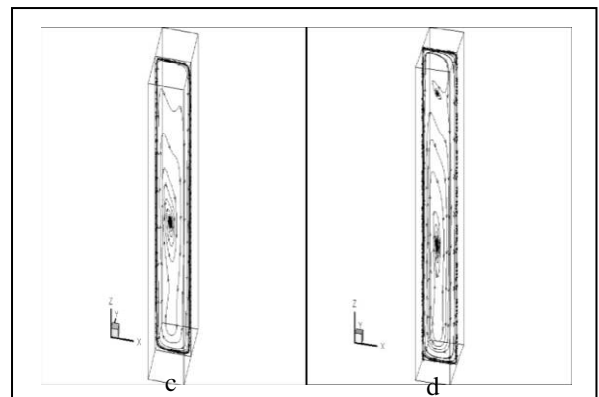
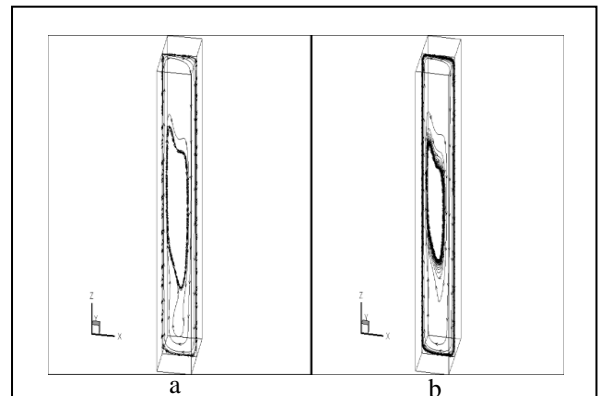


Fig. 4 Thermal fields for the configuration C in the plane $y = D / 2 = 0.36m$ where: (a) $Ra=2.5 \cdot 10^9$, (b) $Ra=6.8 \cdot 10^9$, (c) $Ra=2.5 \cdot 10^{10}$, (d) $Ra=6.8 \cdot 10^{10}$, (e) $Ra= 10^{11}$



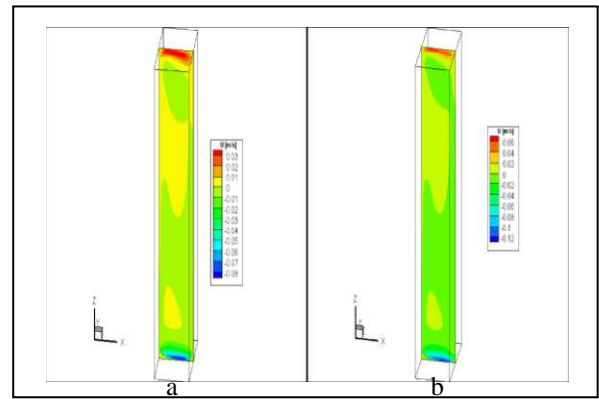
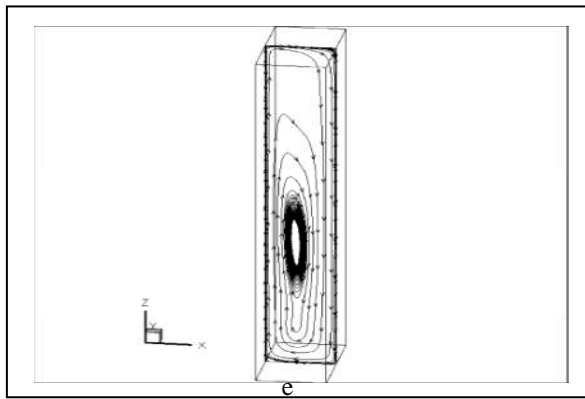


Fig. 5 speed vectors (UW) for configuration C in the plane $y=D/2=0.36\text{m}$ where: (a) $Ra=2.5 \cdot 10^9$, (b) $Ra=6.8 \cdot 10^9$, (c) $Ra=2.5 \cdot 10^{10}$, (d) $Ra=6.8 \cdot 10^{10}$, (e) $Ra= 10^{11}$

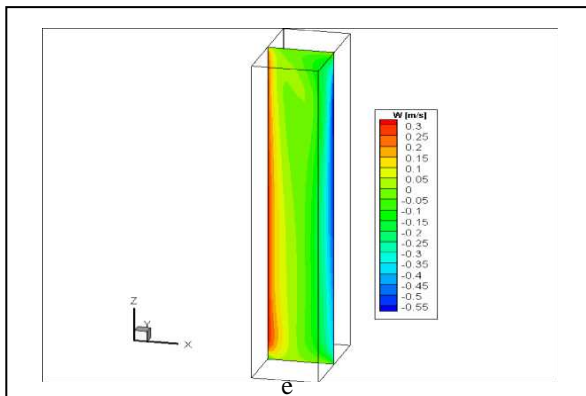
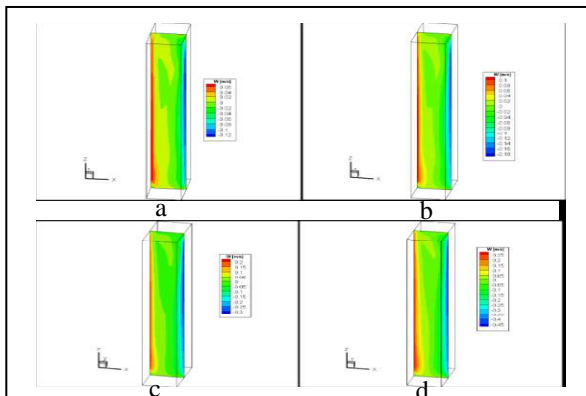


Fig. 6 vertical velocity field for the configuration C in the plane $y = D / 2 = 0.36\text{m}$. where: (a) $Ra=2.5 \cdot 10^9$, (b) $Ra=6.8 \cdot 10^9$, (c) $Ra=2.5 \cdot 10^{10}$, (d) $Ra=6.8 \cdot 10^{10}$, (e) $Ra= 10^{11}$

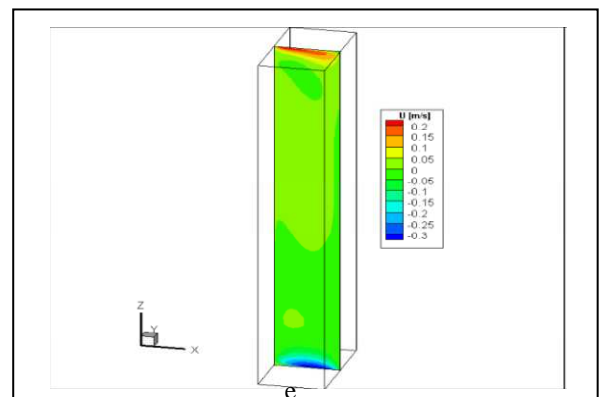
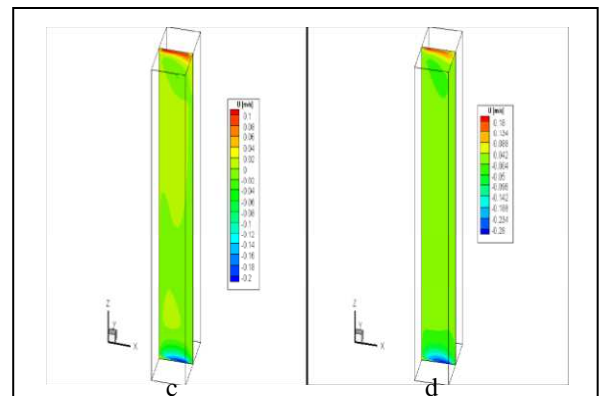


Fig. 7 Champs-speed horizontal configuration C in the plane $y = D / 2 = 0.36\text{m}$. where: (a) $Ra=2.5 \cdot 10^9$, (b) $Ra=6.8 \cdot 10^9$, (c) $Ra=2.5 \cdot 10^{10}$, (d) $Ra=6.8 \cdot 10^{10}$, (e) $Ra= 10^{11}$

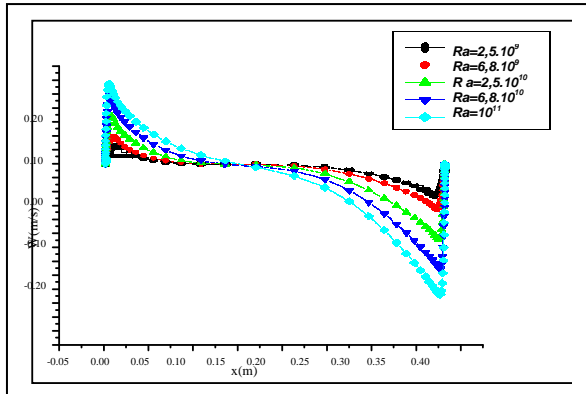


Fig. 8 Profile of vertical velocity for different values of the Rayleigh number in the configuration C with the plane $y = D / 2 = 0.36\text{m}$ and $z = H / 2 = 1.23\text{m}$.

4. Conclusion

In conclusion, the results obtained are in good agreement with experimental and numerical results. We can confirm through this study the influence of Rayleigh number so the ΔT (as fluid characteristics remain unchanged), the convection and consequently trigger zone of turbulent instability. Given the consistency of our results with those established numerically and experimentally by others we were able to validate the calculation for the configurations studied with the $K-\varepsilon$ standard and finite volumes. Therefore, we can look at dice, to address this problem in the future with the introduction of a computer code designed on the method of simulation (Large Eddy Simulation: LES), based on the results obtained in this work.

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