International Journal of Mechanics and Energy (IJME)
Vol. 1, Issue 2, 2013, ISSN: 2286-5845

Entropy Generation in Double Diffusive Convection through a Rectangular Porous Cavity

Ali Mchirgui [1*], Nejib Hidouri[1], Mourad Magherbi[2], Ammar Ben Brahim[1]

[1]*Gabès University, Engineers National School, Chemical and Process Engineering Department, Applied Thermodynamics Unit, Omar Ibn El Khattab Street, 6029 Gabès, Tunisia*

[2]*Gabès University, High Institute of Applied Sciences and Technology, Applied Thermodynamics Unit, Omar Ibn El Khattab Street, 6029 Gabès, Tunisia*

Abstract: Entropy generation in double diffusive convection through a rectangular porous cavity saturated by a binary perfect gas mixture is numerically studied using Darcy – Brinkman formulation. The set of equation describing the phenomenon is solved by using a modified version of the Control Volume Finite-Element Method. Effect of the enclosure geometry on entropy generation was investigated. The results are numerically presented through graphs and maps to observe the effects of aspect ratio of the cavity on entropy generation for the two cases of opposite and cooperatives buoyancy forces.

Key words: Numerical method, cavity, porous medium, heat and mass transfer, entropy generation, double diffusive convection.

1. Introduction

Double diffusive convective flow caused by the combined influence of thermal and solute buoyancy forces through porous media has been frequently studied due to its importance in various technological applications. A large overview of convection in porous media for many systems and situations are well documented in the literature [1, 2]. A numerical study of double-diffusive natural convection in a porous cavity using the Darcy–Brinkman formulation was reported by Goyeau et al. [3]. Kramer et al. [4] used the boundary domain integral method to study double diffusive natural convection in porous media. Khadiri et al. [5] numerically studied thermosolutal natural convection through homogeneous and isotropic porous media, saturated with a binary fluid in two and three dimensional approximations.

Interest in second law analysis has recently been intensified; it is the basis of most formulations of both equilibrium and no equilibrium thermodynamics. Baytas [6] studied entropy generation in natural convection in an inclined enclosure with differentially heated vertical walls and insulated horizontal walls. He found that as Darcy-modified Rayleigh number decreases, heat transfer irreversibility begins to dominate fluid friction irreversibility. Entropy generation due to forced convection in a porous medium was analytically investigated by Hooman et al. [7, 8]. Hidouri et al. [9] and Magherbi et al. [10] numerically studied entropy generation in double diffusive convection in a square cavity by considering the cross thermal diffusion effects.

* **Corresponding Author: Ali Mchirgui**
E-Mail: ali.mchirgui@voila.fr

This study investigates a porous enclosure submitted to double diffusive convection and saturated with a binary perfect gas mixture. The work analyzes the influence of the geometry of the cavity on entropy generation in transient state for both cases of cooperative and opposite buoyancy forces.

2. Problem definition

A two dimensional rectangular porous cavity of height *a* and length *b*, and saturated with a binary perfect gas mixture and submitted to horizontal temperature and concentration gradients is considered. The heated and the cooled vertical walls are at uniform but different temperatures and concentrations (T_h, C_h) and (T_c, C_c), while the two horizontal walls are insulated. A schematic of this model is presented in Fig. 1. The porous medium is isotropic, homogeneous and in thermodynamic equilibrium with the fluid. All physical properties of the fluid are assumed to be constant, except its density which satisfies the Boussinesq approximation such that:

$$\rho(C,T)=\rho_0[1-\beta_T(T-T_0)-\beta_C(C-C_0)] \quad (1)$$

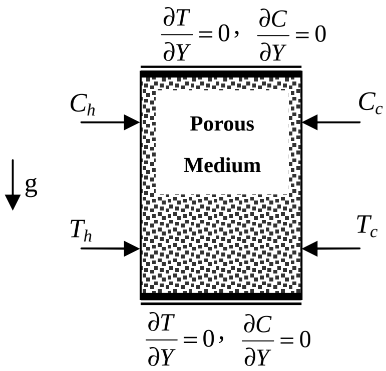

Fig.1 Schematic diagram of the porous enclosure

By using the following dimensionless variables:

$$u=\frac{u^*}{W}\ ;\ v=\frac{v^*}{W}\ ;\ x=\frac{x^*}{a}\ ;\ y=\frac{y^*}{a}\ ;\ \tau=\frac{t.W}{a}\ ;$$

$$p=\frac{P-P_0}{\rho_0 W^2}\ ;\ \theta=\frac{T-T_0}{\Delta T}\ ;\ \phi=\frac{C-C_0}{\Delta C}\quad \Delta T=T_h-T_c\ ;$$

$$\Delta C=C_h-C_c\ ;\ N=\frac{G_{rS}}{G_{rT}}\ ;\ G_{rT}=\frac{g\beta_T\,\Delta Ta^3}{\upsilon^2}\ ;$$

$$G_{rS}=\frac{g\beta_C\,\Delta Ca^3}{\upsilon^2}\ ;\ Pr=\frac{\nu}{W.a}\ ;\ \Lambda=\frac{\mu_{eff}}{\mu}\ ;\ Da=\frac{K}{a^2}\ ;$$

$$\sigma=\frac{(\rho c)_m}{(\rho c)_f}\ ;\ R_K=\frac{k_m}{k_f}\ ;\ Le=\frac{\alpha_f}{D}\ ; \quad (2)$$

The macroscopic conservation equations describing the transport phenomena in the cavity are [1]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (3)$$

$$\frac{1}{\varepsilon}\cdot\frac{\partial u}{\partial \tau}+\frac{1}{\varepsilon^2}div(uU-\Lambda \Pr grad\ u)=-\frac{\Pr}{D_A}u-\frac{\partial p}{\partial x} \quad (4)$$

$$\frac{1}{\varepsilon}\cdot\frac{\partial v}{\partial \tau}+\frac{1}{\varepsilon^2}div(vU-\Lambda Pr\ grad\ v)=-\frac{Pr}{D_A}v-\frac{\partial p}{\partial y}+G_{rT}(\theta+N\phi) \quad (5)$$

$$\sigma\frac{\partial \theta}{\partial \tau}+div(\theta U-R_k grad\ \theta)=0 \quad (6)$$

$$\varepsilon\frac{\partial \phi}{\partial \tau}+div(\phi U-\frac{\varepsilon}{Le}grad\phi)=0 \quad (7)$$

The average heat and mass transfer fluxes at the heated walls are given in dimensionless terms by the Nusselt and Sherwood numbers, respectively:

$$Nu=\int_0^1(-\frac{\partial \theta}{\partial x})dy \quad (8)$$

$$Sh=\int_0^1(-\frac{\partial \phi}{\partial x})dy \quad (9)$$

The dimensionless initial and boundary conditions are:

For the hole space, at $\tau=0$:

$u=v=0$, $p=0$, $\theta=1-x$ and $\phi=1-x$.

At $x=0$: $\phi=\theta=0.5$

At $x=1$: $\phi=\theta=-0.5$

At $y=0$ and $y=1$: $\frac{\partial \theta}{\partial y}=\frac{\partial \phi}{\partial y}=0$ (10)

3. Entropy generation

Following Hidouri et al.[9] and Hooman et al. [7, 8], the expression of the volumetric entropy generation in double diffusive convection through a porous medium for a single diffusing species in 2D approximation is given by:

$$S = \frac{k_m}{T_0^2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right]$$
$$+ \frac{\mu}{T_0.K}(U^2 + V^2) + \frac{\mu}{T_0}\left[2\left(\frac{\partial U}{\partial X}\right)^2 + 2\left(\frac{\partial V}{\partial Y}\right)^2 + \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X}\right)^2\right] \quad (11)$$
$$+ \frac{RD_e}{C_0}\left[\left(\frac{\partial C}{\partial X}\right)^2 + \left(\frac{\partial C}{\partial Y}\right)^2\right] + \frac{RD_e}{T_0}\left[\left(\frac{\partial C}{\partial X}\right).\left(\frac{\partial T}{\partial X}\right) + \left(\frac{\partial C}{\partial Y}\right).\left(\frac{\partial T}{\partial Y}\right)\right]$$

This expression is the result of regrouping three terms: the entropy generation due to thermal gradients, the viscous dissipation and the diffusion entropy generation terms.

The dimensionless form of local entropy generation is obtained by using the dimensionless variables previously listed and is given by:

$$N = N_\theta + N_d + N_f \quad (12)$$

where:

$$N_\theta = \left[\left(\frac{\partial \theta}{\partial x}\right)^2 + \left(\frac{\partial \theta}{\partial y}\right)^2\right]$$
$$N_d = \varphi_1\left[\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2\right] + \varphi_2\left[\left(\frac{\partial \phi}{\partial x}\right).\left(\frac{\partial \theta}{\partial x}\right) + \left(\frac{\partial \phi}{\partial y}\right).\left(\frac{\partial \theta}{\partial y}\right)\right] \quad (13)$$
$$N_f = Br^*\left\{u^2 + v^2 + Da\left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right]\right\}$$

$\varphi_1 = \frac{D_e R}{k.\Omega}\left(\frac{\Omega'}{\Omega}\right)\Delta C$ and $\varphi_2 = \frac{D_e R}{k.\Omega}.\Delta C$ are dimensionless coefficients, called irreversibility distribution ratios. $Br^* = \frac{Br}{\Omega}$ is the modified Darcy-Brinkman number.

$Br = \frac{\mu.W^2.a^2}{k_m.\Delta T.K}$, $\Omega = \frac{\Delta T}{T_0}$, $\Omega' = \frac{\Delta C}{C_0}$ are the Brinkman number, the temperature and the concentration ratios, respectively.

The total dimensionless entropy generation is obtained by numerical integration, over the cavity volume A, of the dimensionless local entropy generation. It is given by:

$$s_T = \int_A s_{lT}.dA\,. \quad (14)$$

4. Numerical procedure

The used numerical method consists on a modified version of the Control Volume Finite-Element Method (CVFEM) of Saabas and Baliga [11] adapted to standard-staggered grids, in which pressure and velocity components are calculated and stored at different points. The SIMPLER algorithm of Patankar [12] is applied to resolve the coupled pressure-velocity equations in order to obtain temperature, concentration and velocity fields at any time τ. Local entropy generation S_{lT} is then calculated at any nodal point of the cavity. The total entropy generation for the entire cavity S_T is then obtained by numerical integration. The used numerical code written in FORTRAN language described and validated in details in Abbassi et al. [13, 14] was modified in order to investigate the present problem.

To test the accuracy of the present numerical study, the average values of Nusselt and sherwood numbers are given in Table 1 and compared with with those of Karmer et al. [4]. It is seen that the results are in good agreement with those given by the literature.

Table 1: Average Nusselt and Sherwood numbers for Le = 10, D_A= 10^{-1}, Ra^* = 100

N		**-1**	**0**	**1**	**2**
Present study	*Nu*	0.98	0.99	1.07	1.08
	Sh	0.98	1.09	2.70	3.01
Kramer et al. [4]	*Nu*	1.0	1.0	1.07	1.09
	Sh	1.0	1.08	2.66	2.95

5. Result and discussion

The considered medium is a rectangular porous cavity with height a and length b (the aspect ratio : A = a/b), filled with a binary perfect gas mixture characterized by Pr = 0.71 and Le = 1.2. The operating parameters are in the following ranges: Da =10^{-2}, Ra^* = 100, $-5 \le N \le 5$ and $0{,}5 \le A \le 5$. All the above parameters are chosen after several numerical computations. Due to large number of parameters, the porous medium proprieties are kept constant, they are given by: $\Lambda = 1$, $\sigma = 1$, $R_k = 1$. The dimensionless coefficients characterizing the entropy generation are: $Br^* = 10^{-4}$, $\varphi_1 = 0.5$ et $\varphi_2 = 10^{-2}$.

Figs. 2 and 3 show the variation of transient entropy generation for different values of aspect ratio.

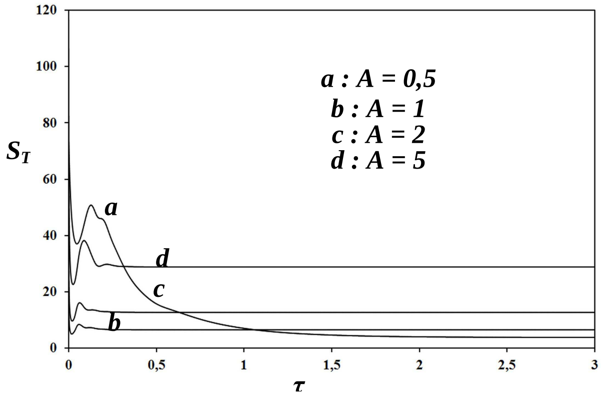

Fig.3: Total entropy generation versus time

$D_A = 10^{-2}$, N = 5

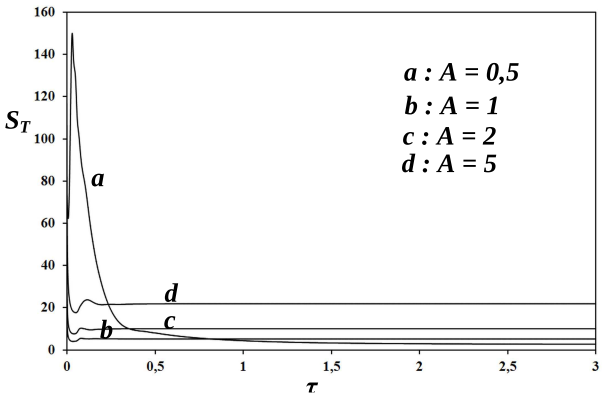

Fig.3: Total entropy generation versus time

$D_A = 10^{-2}$, N = -5

As it can be seen, entropy generation increases at the beginning of the transient state, the heat transfer is made by simple conduction and the mass transfer is made by diffusion mode. It can be noticed that oscillations of entropy generation before the steady state corresponds to the non linear branch of thermodynamics for irreversible processes. Entropy generation decreases and tends towards a constant asymptotic value at the steady state showing that the system's evolution follows the linear branch of thermodynamics for irreversible processes.

In steady state, it is important to notice that entropy generation increases with the aspect ratio for the two cases of opposite and cooperative buoyancy forces. In fact, this can be justified from Figs. 4, 5 and 6 showing the isoconcentration lines, the isotherm lines and the streamlines for three values of aspect ratio.

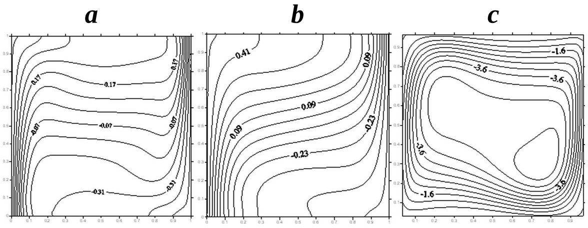

Fig.4: a) Isoconcentrations, b) isotherms, c) streamlines for *A* = 1, *N* = 5.

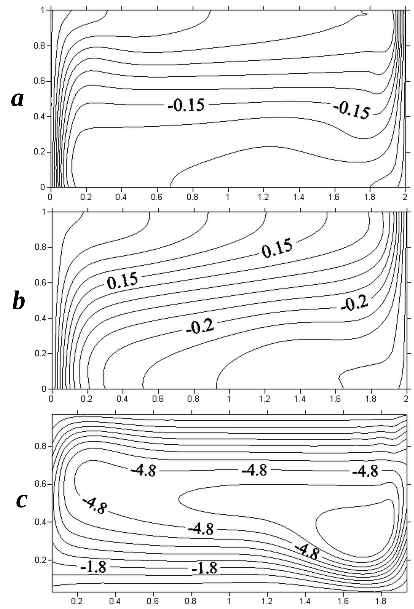

Fig.5: a) Isoconcentrations, b) isotherms, c) streamlines for A = 0.5, N = 5.

It can be seen that heat and mass transfer are more important as the aspect ratio is higher. For *A* = 1 and 0.5, the active walls have the same length but the distance between active walls is higher for A = 0.5 which explains the decrease of heat and mass transfer. For *A* = 5, the length of active wall increases and the heat and mass exchanged between these walls and the porous medium is more important. It is also seen that velocity is intensified when increasing the aspect ratio. As a result, irreversibilities due to heat transfer, diffusion and fluid friction increase with the aspect ratio and then the total entropy generation is more important.

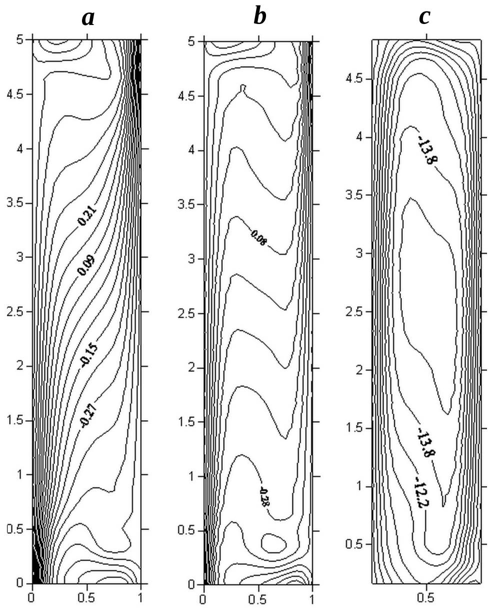

Fig.6: a) Isoconcentrations, b) isotherms, c) streamlines for *A* = 5, *N* = 5.

From Figs. 7, for cooperative buoyancy forces, it is observed that entropy generation is localised on top of cooled and bottom of heated walls, this situation is inversed for opposite buoyancy force. This repartition of local entropy generation is due to the fact that, for the considered value of Darcy number, the entropy generation is mainly due to heat and mass transfer.

5. Conclusion

Influence of aspect ratio on entropy generation in transient state of double diffusive convection through a porous enclosure is numerically studied. It can be concluded that the total entropy generation increases with the aspect ratio. The entropy generation is mainly due to heat and mass transfer for $D_A=10^{-2}$ and it is localised near the active walls.

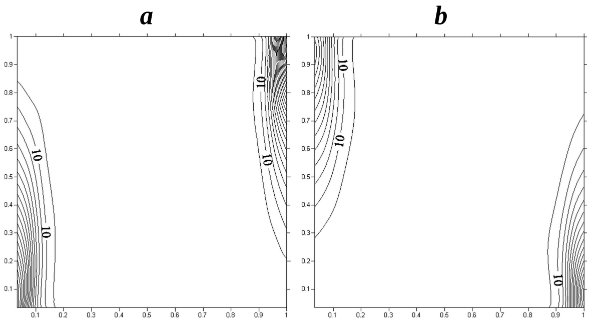

Fig. 7: Entropy generation maps at *A* = 1
***a) N* = 5, *b) N* = -5**

References

[1] D.A. Nield, A. Bejan, Convection in porous Media, 2nd edition, Springer-verlag. New York (1999).

[2] V. Kambiz, Handbook of porous media, 2nd edition, Taylor & Francis, (2005).

[3] B. Goyeau, JP. Songbe, D. Gobin, Numerical study of double diffusive natural convection in a porous cavity using the Darcy Brinkman formulation. Int. J. Heat Mass Trans., 39,(1996) 1363–1378.

[4] J. Kramer, R. Jecl, L. Skerget, Boundary domain integral method for the study of double diffusive natural convection in porous media. Eng. Analysis with Bound. Elements. 31, (2007) 897–905.

[5] A. Khadiri, R. Bennacer, M. Hasnaoui, A. Amahmid, Two- and three-dimensional multiple steady states in a porous cavity heated and salted from below. Int. J. Therm. Sci., 50, (2011) 918-929 .

[6] A.C. Baytas, Entropy generation for natural convection in an inclined porous cavity, Int. J. Heat Mass Trans., 43 (2000) 2089–2099.

[7] K. Hooman, H. Gurgenci, A. A. Merrikh, Heat transfer and entropy generation optimization of forced convection in a porous-saturated duct of rectangular cross-section, Int. J. Heat Mass Trans., 50, (2007) 2051-2059 .

[8] K. Hooman, F. Hooman, S.R. Mohebpour, Entropy generation for forced convection in a porous channel with isoflux or isothermal walls, Int. J. Exergy, 5 (1) (2008) 78-96 .

[9] N.Hidouri, H. abbassi, M. Magherbi, A. Ben Brahim, Influence of thermodiffusion effect on entropy generation in thermosolutal convection, Far East J. App. Math., 25, 2 , (2006) 179 – 197 .

[10] M. Magherbi, N. Hidouri, A. Ben Brahim, Entropy generation optimisation in double diffusive convection considering Soret effect. Adv. and App. in Fluid Mec. 1, 2, (2007) 181 – 205 .

[11] H. J.Saabas, B.R. Baliga, Co-located equal-order control-volume finite-element method for multidimensional incompressible fluid flow. Part I: Formulation. Numer. Heat Transfer Pt. B-Fund. 26, (1994) 381–407.

[12] S.V. Patankar, Numerical heat transfer and fluid flow; In Computational Methods in Mechanics and Thermal Sciences. Hemisphere/Mac Graw-Hill. NY, USA. (1980).

[13] H.Abbassi, S.Turki, S. Ben Nasrallah, Mixed convection in a plane channel with a built-in triangular prison. Numer. Heat Transfer Pt. A-Appl., 39, (2001) 307–320.

[14] H.Abbassi, S.Turki, S. Ben Nasrallah, Numerical investigation of forced convection in a plane channel with a built-in triangular prison. Int. J. Therm. Sci. 40, 2001, 649-658.