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A Numerical Study on the Effect of Radiation on Natural Convection in a Two-Square Duct Annuli Filled with a Semi-Transparent Fluid

Bouanani Mohammed, Benbrik Abderrahmane, Soualmi Rabiaa, Cherifi Mohammed*

Laboratory of Petroleum Equipments Reliability and Materials, Faculty of Hydrocarbons and Chemistry, Université M'Hamed Bougara, Boumerdès, Algérie.

Abstract: In the present study, a numerical investigation of natural convection and volumetric radiation interactions has been analysed in an annulus between two isothermal concentric square ducts filled with a semi-transparent medium. Two-dimensional solution was obtained using a discrete ordinates method based on a combined finite volume-immersed boundary method. The fluid is assumed to be gray absorbing-emitting and non-scattering and all walls are gray, diffuse and opaque. The Rayleigh number is fixed to 10^6, and a study of the effect of three optical thicknesses τ=*0.2-1-5* is performed on the thermal and dynamic fields and consequently on the heat transfer rate has been examined. A pure natural convection case has been shown for a comparative study. It is found that the radiation has a large influence on the flow and surface averaged Nusselt number and especially for high Rayleigh numbers.

Keywords:natural convection, participating medium, volumetric radiation, immersed boundary, discrete ordinates method, square ducts.

1. Introduction

Over the past years, natural convection in complex geometries including enclosures with obstacles has been a fundamental problem in heat transfer due to its wide applications. These applications include oil storage tanks, heat exchangers, solar collectors, cooling electronic components, etc. Among the most famous complex geometries are enclosures with a circular cylinder. However, many researchers focused on the other shapes as the square bodies [1-3] or triangular cylinder as Rath et al. [4] who investigated numerically the natural convection in a rectangular enclosure within isothermal horizontal cylinder of circular, square and triangular shapes. Enclosures within square cylinders was a challenge in the study of natural convection in complex geometries for many researchers. House et al. [3] were the first to study the natural convection in a differentially heated enclosure with a square inner body. They studied the effect of two parameters of a square heat conducting body which are the body size and the ratio of thermal conductivity of the body to that of the fluid in an enclosure with differentially heated sidewalls, for a Rayleigh number 10^3 to 10^6 and a wide range of Prandtl number. They found that the heat transfer may be enhanced (reduced) as the conductivity ratio gets less (greater) values and may attain a minimum as the body size increases. Many studies focused on the same configuration with different thermal boundaries or by adopting an angle of inclination to the enclosure [5-6]. On the other hand few studies have investigated the volumetric radiation effect in complex geometries despite the large number of researchers that have focused on its significant effect on the natural convection in simple enclosures [7-10]. Lari et al. [8] have found that the volumetric radiation cannot be neglected even for low temperature differences by studying its impact on the dynamic and thermal fields in a square enclosure under normal room conditions. Han and Baek [11] have investigated thenatural convection and volumetric radiation interaction in a concentric and eccentric horizontal cylindrical annuli with a large temperature difference.

* **Correspondingauthor:**Bouanani Mohammed
E-mail: m.bouanani@univ-boumerdes.dz

Zhang et al. [12] have studied the effect of radiation on natural convection in a square enclosure heated by a circular cylinder and filled with semi-transparent medium. They analyzed the effect of several parameters as Rayleigh number, temperature difference and optical thickness. It was found that the radiation cannot be neglected due its significant impact on the flow and heat transfer rate, especially at high Rayleigh number and shown that the radiation heat transfer is affected obviously much more than the convective one by optical thickness or temperature difference variations.

From the above literature survey, it has been common practice to neglect the volumetric radiative heat transfer effect in enclosures within square cylinder. The main purpose of this paper is to examine the contribution of an absorbing-emitting and non-scattering fluid to the natural convection in that configuration. Therefore, an in-house code has been developed using a combined finite volume-immersed boundary method to obtain the velocity and temperature fields in addition to a discrete ordinates method to solve the radiative transfer equation. A complete parametric study is made for a variety of Rayleigh numbers and optical thicknesses.

2. Problem Statement

The physical model proposed for this study is a two dimensional annulus between two isothermal ducts. The outer and inner square ducts are maintained at two different temperatures T_c and T_h respectively with $T_h > T_c$ with side length of L and $0.3L$ respectively as shown in Figure 1. The flow is assumed to be steady, laminar and incompressible. All the walls are gray, diffuse and opaque. The working fluid (air) is assumed to be Newtonian, gray, absorbing-emitting and non-scattering with constant physical properties except the density in the buoyancy term which follows the Boussinesq approximation.

3. Materials and Methods

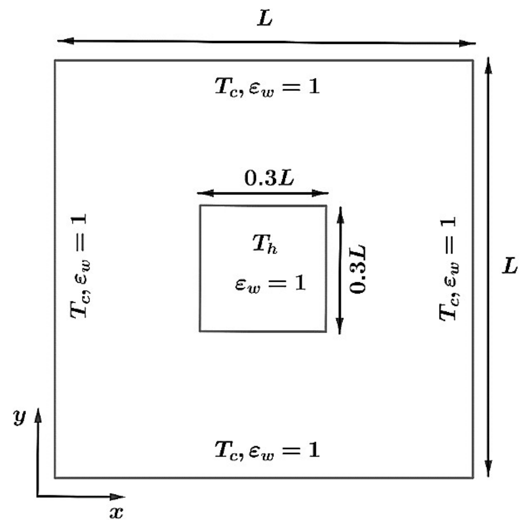

Fig. 1 Schematic geometry of physical problem.

3.1 Governing equations

With the assumptions above, the governing equations describing the flow and thermal fields in the present study are the continuity, momentum and energy equations in their non-dimensional forms defined as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}-q=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial P}{\partial x}+Pr\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+f_x \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial P}{\partial y}+Pr\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) +RaPrT+f_y \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+S_R+f_T \tag{4}$$

where q, f_x, f_y and f_T are the mass source/sink and the forcing terms to satisfy the mass conservation, the no-slip condition and the thermal boundary condition forcing term respectively for the cell containing the immersed boundary as described by Kim et al. [13].

The dimensionless variables in these equations are defined as:

$$x=\frac{x^*}{L},\ y=\frac{y^*}{L},\ u=\frac{u^*}{\alpha L},$$

$$v=\frac{v^*}{\alpha L},\ P=\frac{P^* L^2}{\rho\alpha^2} \text{ and } T=\frac{T^*-T_0^*}{T_h^*-T_c^*}$$

The superscript * denotes dimensional variables. The dimensionless parameters are defined as follows:

$$Ra = \frac{g\beta L^3\left(T_h^* - T_c^*\right)}{\nu\alpha} \text{ and } Pr = \frac{\nu}{\alpha}$$

withS_R is the radiative source term calculated by solving the radiative transfer equation, as it will be described in the next section.

3.2Radiation model

The traveling radiation intensity I for a gray, absorbing-emitting and non-scattering is described by the dimensionless radiative transfer equation RTE as follows:

$$\mu\frac{\partial I}{\partial x} + \xi\frac{\partial I}{\partial y} + \tau I = \tau I_b \qquad (5)$$

where $\tau = \kappa L$ is the optical thickness where κ is the absorption coefficient. I_b is the black body emission at local temperature, calculated as follows:

$$I_b = \frac{1}{4\pi}\left(1 + \frac{T}{\Theta_0}\right)^4 \qquad (6)$$

The radiative boundary conditions on all the walls are expressed as follows:

$$I\left(x,y,\vec{\Omega}\right) = \frac{\varepsilon_w}{4\pi}\left(1 + \frac{T}{\Theta_0}\right)^4 + \frac{(1-\varepsilon_w)}{\pi}\times \left(\int_{\vec{n}.\vec{\Omega}'<0}\left|\vec{n}.\vec{\Omega}'\right| I\left(x,y,\vec{\Omega}'\right)d\Omega'\right) \qquad (7)$$

where$\vec{\Omega}$presents the leaving direction while$\vec{\Omega}'$presents the incident one. The emissivity of walls ε_w is equal to unity as all walls are assumed to be gray, diffuse and opaque.

The discrete ordinate method DOM is used to solve the equation (5) for a finite set of directions spaning the total solid angle4π. Equation (5) can be written for each discrete direction$\vec{\Omega}_m$as follows:

$$\mu_m\frac{\partial I_m}{\partial x} + \xi_m\frac{\partial I_m}{\partial y} + \tau I_m = \tau I_b \qquad (8)$$

Using DOM, the radiation intensity is calculated for each direction in every position of the fluid then the incident radiation G and the radiative heat flux Q_R are calculated using the expressions below:

$$G = \int_{4\pi} I d\Omega = \sum_{m=1}^{M} w_m I_m \qquad (9)$$

$$Q_R = \int_{4\pi}\vec{\Omega} I d\Omega = \sum_{m=1}^{M} w_m I_m \vec{\Omega}_m \qquad (10)$$

$$S_R = -\frac{\Theta_0}{Pl}\vec{\nabla}.\overrightarrow{Q_R} \quad (11)$$

$$\vec{\nabla}.\overrightarrow{Q_R} = \tau\left(4\pi I_b - G\right) \quad (12)$$

3.3Boundary conditions

The no-slip and temperature boundary conditions are as follows:

On all fluid/solid interfaces: $u = v = 0$

On the outer duct: $T = T_c = -0.5$

On the inner duct: $T = T_h = 0.5$

3.4Heattransfer

The heat transfer in the enclosure is quantified by the wall surface-averaged Nusselt numbers (convective, radiative and total) which are defined respectively by:

$$\overline{Nu}_{con} = \frac{1}{4}\left[\int_0^L \left|\frac{\partial T}{\partial x}\right|_{x=0,L} dy + \int_0^L \left|\frac{\partial T}{\partial y}\right|_{y=0,L} dx\right] \qquad (13)$$

$$\overline{Nu}_{rad} = \frac{\Theta_0}{4Pl}\varepsilon_w \int_0^L \left[\frac{1}{4}\left(1+\frac{T}{\Theta_0}\right)^4 - \int_{\vec{n}.\vec{\Omega}'<0}\left|\vec{n}.\vec{\Omega}'\right| I\left(x,y,\vec{\Omega}'\right)d\Omega'\right]_{x=0,L} dy + \frac{\Theta_0}{4Pl}\varepsilon_w \int_0^L \left[\frac{1}{4}\left(1+\frac{T}{\Theta_0}\right)^4 - \int_{\vec{n}.\vec{\Omega}'<0}\left|\vec{n}.\vec{\Omega}'\right| I\left(x,y,\vec{\Omega}'\right)d\Omega'\right]_{y=0,L} dx \qquad (14)$$

$$\overline{Nu}_t = \overline{Nu}_{con} + \overline{Nu}_{rad} \quad (15)$$

4. Materials and Methods

Numerical solution of the governing equation is obtained by a finite volume method using the SIMPLE algorithm on a staggered uniform grid. The convective term is discretized using a power law scheme and the central difference scheme is used for the diffusive term. The immersed boundary method is used to handle the square shaped cylinder to impose accurately the no-slip condition on the fluid-solid interface. Discrete ordinates method with S_8 quadrature is used to define the radiative source term. The algebraic resulting system is solved iteratively until satisfying the following convergence criterion:

$$\left|\frac{\sum_{i,j}\left|\phi_{i,j}^{n}-\phi_{i,j}^{n-1}\right|}{\sum_{i,j}\left|\phi_{i,j}^{n}\right|}\right|<10^{-6} \quad (16)$$

4.1Grid independency

A grid refinement is performed for two optical thicknesses to obtain an independent solution of the grid size. Therefore, 100×100 grid size has been used for the computations to be reported in this study.

4.2Validation

An extensive validation has been performed by simulating three different cases. In this work two cases are shown. The first was natural convection in a differentially heated enclosure within a centric square heat conducting body for different body sizes. Table 1 shows that 0.05% is the maximum deviation of the surface-averaged Nusselt number compared to the results of House et al. [3] for two thermal conductivities ratio.

Table 1Validation of Nu for Ra=10^6 with published results.

Thermal conductivity ratio	Present work	House et al.[3]	Deviation %
$k_s / k_f = 0.2$	4.625	4.624	0.02
$k_s / k_f = 5$	4.327	4.325	0.05

In order to validate the radiation code, another test has been performed. Table 2 shows the total surface-averaged Nusselt number of differentially heated enclosure with two horizontal adiabatic walls for Ra=5.10^6 and three different optical thicknesses.

Table 2 Total surface-averaged Nusselt number for Ra=5.10^{6}published results.

Optical thickness	$\tau = 0.2$	$\tau = 1$	$\tau = 5$
Present work	46.22	38.98	31.83
Yücel et al. [10]	46.11	38.93	31.76

5. Results and Discussion

As mentioned above, in this study, the effect of optical thickness 0.2, 1 and 5 on the combined radiation-natural convection heat transfer and the flow for a fixed Rayleigh number Ra=10^6 is reported. In

Pure convection ***τ = 0.2*** ***τ = 1*** ***τ = 5***

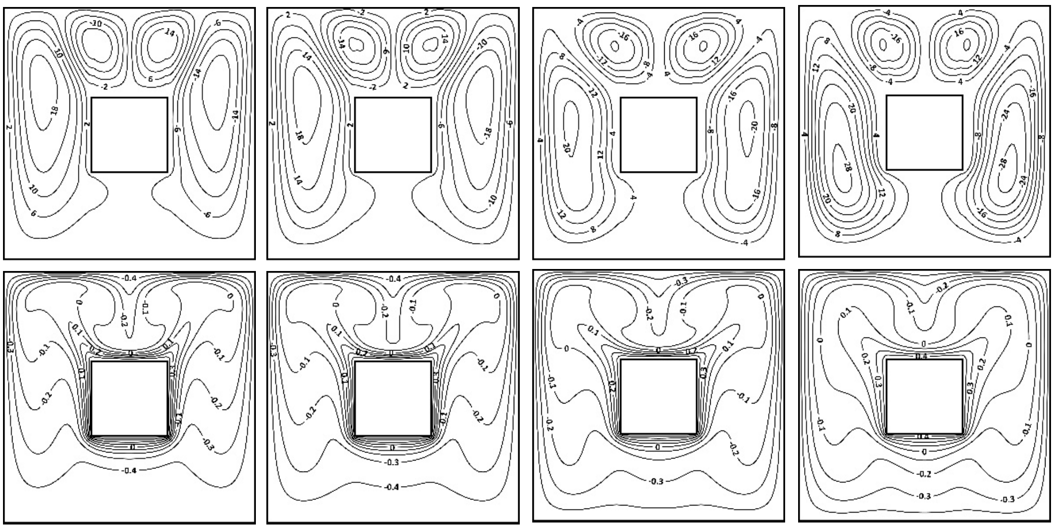

Fig. 2 Streamlines and isotherms at *Ra=10^6* for different optical optical thicknesses.

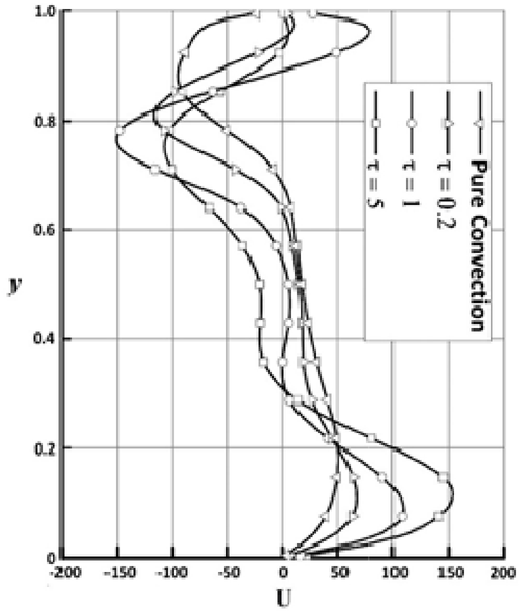

Fig.3Horizontal Velocity profile for different optical thicknesses when*Ra=10*^{6}

addition, the horizontal velocity and the heat transfer rate in terms of Nusselt number is analyzed. Planck and Prandtl numbers have been taken constants for the entire simulation with *Pl=0.02* and *Pr=0.71* respectively.

5.1Basic flow and isotherm features

Fig.2 shows streamlines and isotherms contours for different optical thicknesses when $Ra=10^{6}$. The buoyancy effect is more intensive than the viscous forces thus the convection mode of heat transfer dominates. Consequently, the vortices intensity is strong which creates an expected flow separation. The fluid at the lateral passage absorbs heat from the hot inner duct where a classical convection is induced between the two vertical walls and on the other hand a Rayleigh-Bénard convection is located at the horizontal top passage. This mixture and competitiveness of the two convection modes create a diagonal plume as described above where the fluid moves diagonally and a separation of the flow is created when hitting the top corner which explains also the inverted plume shape. t can be noticed a slim

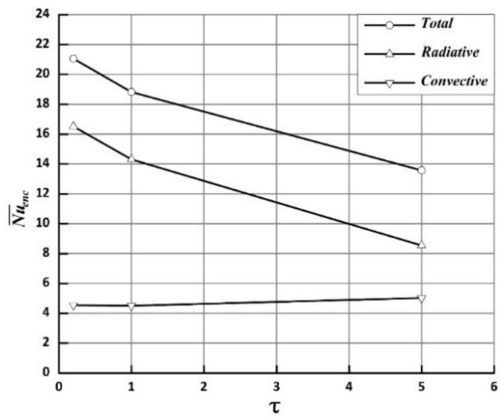

Fig.4 Surface-averaged Nusselt numbers for different optical thicknesses when*Ra=10*^{6}

plume shape due to the strong convective forces. Fig.2 shows also that the fluid at the lower passage is colder comparatively to the lower Rayleigh numbers (not shown). The plume at the top passage becomes larger with the optical thickness as the fluid absorbs more heat thus the secondary vortices are stretched horizontally which pushes the cores of the main vortices downward and this leads to heat the lower passage in addition to the absorption of heat due to the opacity of the medium which is called the far reaching effect Han and Baek [11].

5.2Velocity

Fig.3 presents the horizontal velocity profile along the *y*-direction for the pure natural convection case and different optical thicknesses when $x=0.2$ and $Ra=10^{6}$. When radiation occurs and optical thickness increases it has been noticed in the previous section that the cores of the main eddies are shifted downward and this explains the enhancement of the horizontal velocityat the lower passage and its reduction between the differentially heated walls where it changes the direction for an optically thicker medium $\tau=5$, as the cores of the eddies become located at the lower half of the enclosure. At the lateral passage, it can be noticed, that the horizontal velocity is not very important which indicates a unidirectional flow due to the convective forces, especially when $\tau < 5$ and it decreases with the opacity of the medium in general. The horizontal velocity independently to the opacity of the medium increases intensively at the top corner due to the diagonal plume which pushes the fluid laterally. The fluid with the high opacity changes its direction early at a lower height due to the same effect of the diagonal plume slope or the flow separation position. However, when $\tau=5$, the behavior is different because the diagonal plume changes two aspects: slope and width.

5.3Heat transfer

Fig.4 shows the effect of optical thickness on the convective, radiative and total surface averaged Nusselt number. It can be observed in Fig.4 that the heat transfer rate decreases with the optical thickness due to the fluid resistance to radiation penetration as described above. However, the convective heat transfer has not been affected by the opacity of medium.

6. Conclusions

This study investigated numerically a two-dimensional combined natural convection-volumetric radiation in a two-square duct annuli filled with a semi-transparent fluid. A combined finite volume and immersed boundary method has been used to solve the continuity and momentum equations in addition to the discrete ordinates method to solve the radiative transfer equation. The distribution of streamlines, isotherms and the Nusselt number was obtained for a fixed Rayleigh number $Ra=10^6$ and three different optical thicknesses 0.2, 1 and 5. The following conclusions are made:

- The effect of Radiation on streamlines, isotherms structure and heat transfer is very important.
- The opacity of the medium decreases the heat transfer rate while it increases and homogenizes the temperature of the fluid.

Acknowledgments

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