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Prandtl number effect on mixed convection inside a horizontal lid-driven rectangular cavity

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Abstract: Mixed convection within a rectangular cavity confining a Newtonian fluid is studied analytically and numerically in the case where the horizontal walls, whose upper one is moving, are adiabatic, whereas those vertical are submitted to uniform density of flux. Numerical calculations are carried out for values of governing parameters within the ranges, $0.1 \le Ri \le 10^3$, $0.1 \le Re \le 10$, $0.1 \le Pr \le 50$ and A = 20, where Ri is the Richardson number, Re is the Reynolds number, Pr is the Prandtl number and A is the aspect ratio. In such a situation, the obtained results testify of a strong influence of Pr on flow and heat transfer characteristics.

Key words: Heat transfer; mixed convection; lid-driven rectangular cavity.

1. Introduction

Mixed convection heat transfer in lid-driven enclosures is one of the most widely studied problems in thermo-fluids area, in view of the number of the works dealing with. This is because the driven cavity configuration is encountered in many practical engineering and industrial applications such as food processing, float glass production [1], crystal growth, flow and heat transfer in solar ponds [2], lubrication technologies [3] and so on. The interaction of the shear driven flow due to lid motion and the natural convective flow caused by buoyancy effect is quite complex, which necessitates a comprehensive analysis to understand the physics of the resulting flow and heat transfer mechanism. In this respect, different configurations and combinations of thermal and dynamical boundary conditions have been considered and analysed by many investigators.

However, the works on the phenomenon have often targeted the square cavity than the rectangular one, which may reveal something different as reported in the study conducted in the same way by Lamsaadi et al. [4], where all the walls are motionless. In fact, these authors have observed a parallelism and stratification for the flow and temperature fields, respectively, from a threshold value of the aspect ratio. In addition to that, the Prandtl number has been seeing without effect on flow and heat transfer, due to the strong domination of the momentum diffusion on the thermal one. In contrast, in the study performed by Moallemi and Jang [5] the Prandtl number seems to affect strongly mixed convection heat transfer in a lid-driven square cavity heated from below and cooled from the top at constant temperatures.

Therefore, the present study addresses this aspect for a horizontal rectangular cavity where mixed convection is subsequent to an upper wall uniformly moving and vertical walls submitted to uniform density of heat flux in the direction of the wall motion. Also, a particular interest is given to the Prandtl number effect on mixed convection heat transfer.

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2. Mathematical formulation

The studied configuration is sketched in Fig. 1. It is a shallow rectangular enclosure of height H' and length L', filled with a Newtonian fluid. The long horizontal walls are adiabatic, while the vertical short ones are submitted to a uniform density of heat flux, q'.

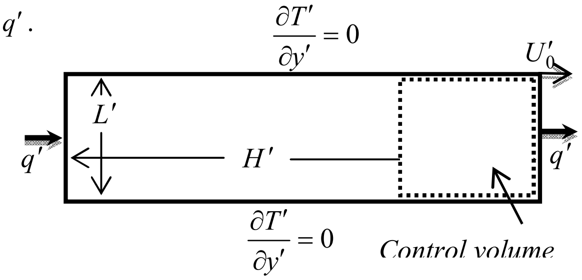

Fig. 1 Schematic view of the geometry and coordinates system.

All these boundaries are rigid, impermeable and motionless apart from the top one which moves in the direction of q' at a constant velocity U'_0,. The main assumptions made here are those commonly used in mixed convection problems [5]. Therefore, using the characteristic scales H', $\rho U_0'^2$, H'/U'_0, U'_0, and $q'H'/k$, corresponding to length, pressure, time, velocity and temperature, respectively, the dimensionless governing equations describing mass, momentum and energy conservation, written in terms of velocity components (u, v), pressure (p) and temperature (T), are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + RiT \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{Pr\,Re}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

To close the problem, the following appropriate boundary conditions are applied

$$u = v = \frac{\partial T}{\partial x} + 1 = 0 \quad \text{for} \quad x = 0 \text{ and } A \tag{5}$$

$$u = v = \frac{\partial T}{\partial y} = 0 \quad \text{for} \quad y = 0 \tag{6}$$

$$u - 1 = v = \frac{\partial T}{\partial y} = 0 \quad \text{for} \quad y = 1 \tag{7}$$

In addition, to analysis the flow structure, the stream function, ψ, related to the velocity components via $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ ($\psi = 0$ on all boundaries) is used.

In the above equations appear the following governing dimensionless parameters:

$$A = \frac{L'}{H'}, \quad Pr = \frac{\nu}{\alpha}, \quad Re = \frac{U'_0 H'}{\nu} \quad \text{and} \quad Ri = \frac{g\beta q' H'^2}{k U_0'^2} \tag{8}$$

which are the aspect ratio of the enclosure, the Prandtl, the Reynolds and the Richardson numbers, respectively.

On the other hand, according to [4], the mean Nusselt number, expressing the overall heat transfer through the fluid-filled cavity, is defined as

$$\overline{Nu} = \int_0^1 -\frac{1}{(\partial T/\partial x)_{x=A/2}}\, dy \tag{9}$$

3. . Numerical approach

Equations (1)-(4) associated with (5)-(7) have been solved using a finite volume method and SIMPLER algorithm in a staggered uniform grid system [6]. The convergence has been considered as reached when $\sum_{i,j}\left|f_{i,j}^{k+1} - f_{i,j}^{k}\right| < 10^{-5}\sum_{i,j}\left|f_{i,j}^{k+1}\right|$, where $f_{i,j}^{k}$ stands for the value of u, v, p or T at the k^{th} iteration level and grid location (i, j) in the plane (x, y). In the limit of the values selected for Pr, Re and Ri, a uniform grid of 200×60 has been judged sufficient to model accurately the flow and temperature fields within a cavity of A =

20 (found as the lower value of A beyond which mixed convection heat transfer does not change).

4. Parallel flow approach

On the basis of Fig. 2, displaying streamlines (left) and isotherms (right) for A = 20, Re = 1, Ri = 100 and various values of Pr, the following simplifications, in the central part of the cavity, can be made:

$$u(x,y) = u(y)\,,\ v(x,y) = 0\,,\ \psi(x,y) = \psi(y) \quad \text{and} \quad T(x,y) = C(x - A/2) + \theta(y) \tag{10}$$

where C is unknown constant temperature gradient in the x-direction.

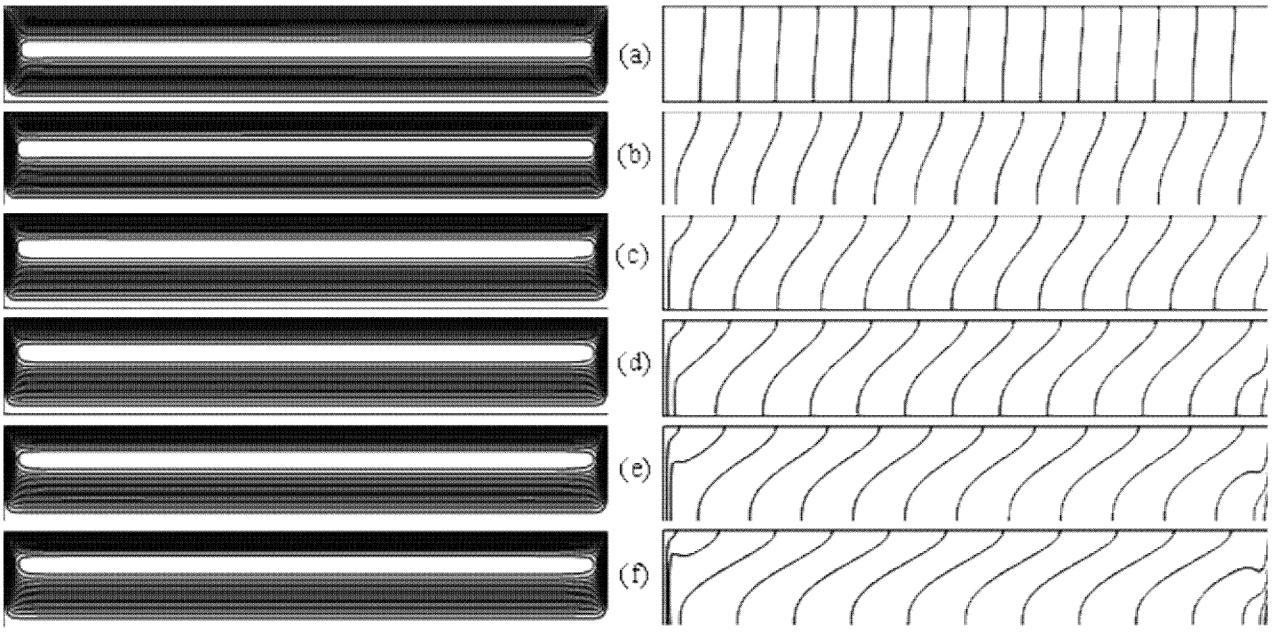

Fig.2 Streamlines (left) and isotherms (right) for Re = 1, Ri = 100 and various values of Pr ((a) Pr = 1, (b) Pr = 10, (c) Pr = 20, (d) Pr = 30, (e) Pr = 40 and (f) Pr = 50).

This leads to the following ordinary non dimensional governing equations:

$$\frac{d^3u}{dy^3} = Re\ Ri\,\frac{\partial T}{\partial x} = Re\ RiC$$

$$\frac{1}{Pr\,Re}\frac{d^2\theta}{dy^2} = Cu \tag{11}$$

with

$$u - 1 = \frac{d\theta}{dy} = 0 \ \text{ for } \ y = 0 \ \text{ and } 1,$$

$$\int_0^1 u(y)dy = 0 \quad \text{and} \quad \int_0^1 \theta(y)dy = 0 \tag{12}$$

as boundary, return flow and mean temperature conditions, respectively.

Using such an approach, the solutions of (11), satisfying (12), are

$$u(y) = \frac{1}{12}Re\,RiC\left(2y^3 - 3y^2 + y\right) + \left(3y^2 - 2y\right) \tag{13}$$

$$\theta(y) = \frac{1}{12}Pr\,Re^2\,RiC^2\left(\frac{y^5}{10} - \frac{y^4}{4} + \frac{y^3}{6} - \frac{1}{120}\right)$$
$$Pr\,ReC\left(\frac{y^4}{4} - \frac{y^3}{3} + \frac{1}{30}\right) \tag{14}$$

The expression of $\psi(y)$ can be deduced from $u = \frac{d\psi}{dy}$ (ψ = 0 for y = 0 or 1), which gives:

$$\psi(y) = \frac{1}{12}Re\,RiC\left(\frac{y^4}{2} - y^3 + \frac{y^2}{2}\right) + \left(y^3 - y^2\right) \tag{15}$$

Therefore, the flow intensity, which corresponds to the maximum value of $|\psi(y)|$ in the central vertical section of the enclosure ($x = A/2$), is $\psi_c = |\psi|_{\max}$.

On the other hand, according to Bejan [7], the energy balance in x-direction is

$$\int_0^1 -\frac{\partial T}{\partial x} dy + Pr\,Re \int_0^1 uT dy = \int_0^1 -\left(\frac{\partial T}{\partial x}\right)_{x=0 \text{ or } A} dy \quad (16)$$

In the parallel flow region and with the application of (10), (16) becomes

$$-C + Pr\,Re \int_0^1 u\theta\, dy = 1 \quad (17)$$

which, when substituted to (13) and (14), gives

$$1 + C + Pr^2\,Re^2 \left(\frac{C}{105} - \frac{Re\,Ri C^2}{3360} + \frac{Re^2\,Ri^2 C^3}{362880} \right) = 0 \quad (18)$$

whose solution, via Newton-Raphson method, for given *Pr*, *Re* and *Ri*, leads to *C*.

Finally, taking into account of (9) and (10), the mean Nusselt number becomes

$$\overline{Nu} = -\frac{1}{C} \quad (19)$$

5. Results and discussion

Mixed convection problem inside the considered cavity is governed by the parameters expressed by (8). Trial calculations, performed gradually with various values of *A*, have shown that, from *A* = 20, the effect of the edge sides disappears so that the flow and temperature fields keep the same configuration in the central part of the cavity, which justifies the choice of this value of *A* and reduces, consequently, the number of the governing parameters to *Pr*, *Re* and *Ri*.

Typical examples of streamlines (left) and isotherms (right) are presented in Fig.2, for *Re* = 1, *Ri* = 100 and various values *Pr*. First of all, it is interesting to observe that the flow is unicellular and clockwise, as a result of cooperating aspect of buoyancy and shear effects, which act together from left to right. Also, as mentioned before, except for the end sides where the flow undergoes a rotation of 180°, this one is parallel to the horizontal boundaries and the temperature is linearly stratified in the horizontal direction, which proves the existence of an analytical solution to the problem. In addition to that, the symmetry of the flow, observed for a dominant buoyancy effect, is generally broken by the shear one. On the other hand, as shown in such a figure, the effect of *Pr* does not clearly appear from the streamlines, as these ones remain identical to themselves, but the isotherms seem to be more affected by this parameter, since their inclination with respect to the vertical direction increases importantly with *Pr*. This expresses a pseudo-conductive regime (quasi-vertical isotherms) that appears at low value of *Pr* and tends to disappear, little by little, with *Pr* to benefit a convective one (inclined isotherms).

Of course, the above results depend on the values of *Re* and *Ri* so that the pseudo-conductive regime cannot exist while increasing these parameters, but *Pr* can keep its effect whatever the values of *Re* and *Ri*, as can be seen from Fig.3 displaying the variations of the flow intensity ψ_c (left) and the heat transfer rate (right) with *Pr* for various values of *Re* and *Ri*. In fact, an augmentation of *Pr* leads to:

(i) a diminution of ψ_c , due to a dominating viscosity effect, which tends to slowdown the fluid motion;

(ii) an augmentation of $\overline{Nu}$, owing to the reduction of the thermal layer boundary thickness near the thermally active vertical walls.

Note that these tendencies depend notably on the values of *Re* and *Ri*.

Another finding is that a perfect agreement between numerical and analytical results is obtained in the limit of the considered values of the governing parameters, which testifies of the validity of the developed parallel flow approximate analytical solution.

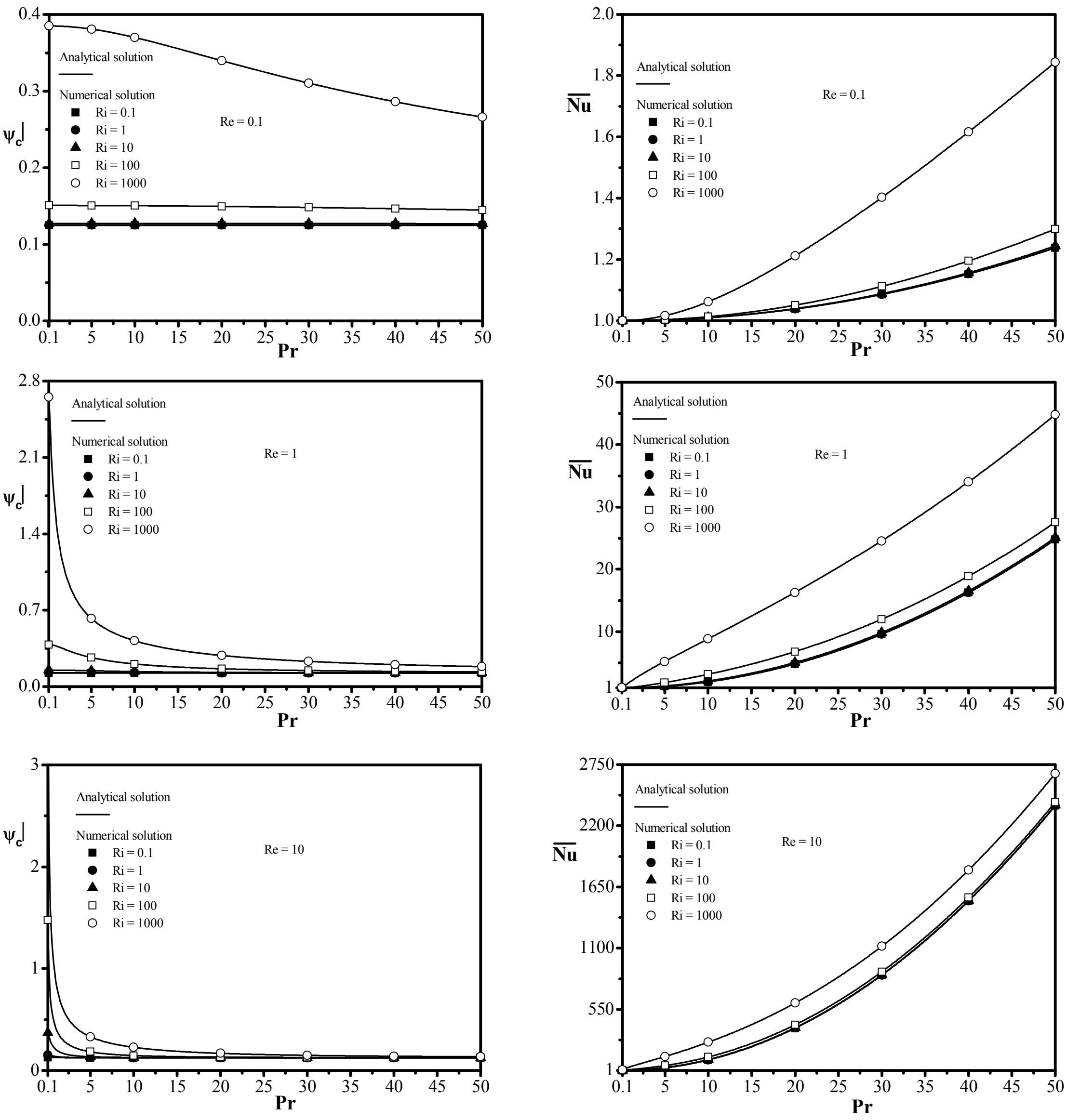

Fig.3 Evolution of ψ_c and $\overline{Nu}$ with *Pr* for various values of *Re* and *Ri*.

6. Conclusion

A numerical and analytical study of mixed convection in a shallow lid-driven rectangular cavity confining a Newtonian fluid is performed in the case where the horizontal walls are adiabatic whereas the vertical ones are submitted uniformly to a density of heat flux oriented in the direction of the ceiling uniform motion.

Unlike buoyancy-driven convection, where the Prandtl number is almost without role, the present results show that this parameter influences significantly the thermo-hydraulic characteristics of the studied configuration. Besides, a good agreement is seen between the numerical and the parallel flow approaches adopted, validating, thus, each other.

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