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Effect of viscous dissipation on thermally developing laminar forced convection for a pseudoplastic fluid

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Abstract: In this work, we treat the thermal development problem, for a pseudoplastic fluid in a single pipe. A fully developed flow is supposed at the pipe inlet, with an imposed temperature at the surface in the case of a heating and cooling. In addition, the effect of viscous dissipation is considered. Finite difference method with an implicit scheme is used to solve the energy equation.

The main objective of the work is to provide results, enabling to well understand the effect of viscous dissipation associated with that of rheological behavior. For this, different values of the Brinkman (*Br*) number characterizing the heat generation by viscous friction, and the rheological index (*n*) have been taken in heating situation as well as cooling. It has been found that the fluid shear-thinning ($n\downarrow$) significantly reduces the dissipative effect, by reducing the friction between the fluid layers.

Keywords: Forced convection; thermal development; pseudoplastic fluid; viscous dissipation.

1. Introduction

In a viscous fluid flow, there is always a friction between the fluid layers (shear). In some cases, this friction is considerable and provides a generation of significant heat within the flow. This heat generation is considered as an internal heat source, which changes the temperature distribution in the medium and thus, the coefficient of heat transfer (*Nu*). The effect of viscous dissipation is usually represented by the Brinkman number (*Br*), which also depends on the heating conditions.

Today, viscous dissipation is very exploited in industry. One can cite plastic polymers extrusion as an example. Originally powder; they are introduced into the extruder, equipped with a screw (one or more). By turning, a highly significant shear is applied to the polymer (particles of powder) which begins to collapse under the effect of heat generated by viscous dissipation. Slowly along the extruder, the temperature increases in the polymer and reaches that of its fusion. Thereafter the molten polymer passes to the molding phase.

From the above, the Graetz problem taking into account the viscous dissipation has interested many authors. Among them, in reference [1], authors studied the Graetz problem in a simple cylindrical pipe and between two parallel plates, for an imposed heat flux. The fluid was considered non-Newtonian, modeled by the power-law. The main study objective is to determine the Nusselt number. Three values of the rheological index were taken (n=1/3, n=1.0, n=3.0). A comparison with the asymptotic solution given in reference [2] shows a good agreement. For a cylindrical pipe, the problem was studied for three heating types in reference [3]: A decreasing flux until zero value for an infinite length, a decreasing flux until a positive value, and then an increasing to infinity flux when length tends to infinite. The author found that the value of the Nusselt tends to zero for the first case. It depends on the value of n and the reference Brinkman number (Br_∞) for the second case. And depends on n and a new dimensionless number β for the third case. For an elliptical section at an imposed temperature, a numerical work based on DADI scheme (Dynamic Alternating Direction Implicit) has been done in reference [4]. The Nusselt number is graphically represented for different values of *Br* and β (ellipse radii ratio=a/b). Authors showed that the value of fully developed *Nu* does not depend on *Br*, but it increases with β. Reference [5] analyzed for a developed dynamic regime in a cylindrical pipe and a Newtonian fluid, the effect of viscous

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increase of the parietal velocity gradient with decreasing *n*. This is due to the decrease of the fluid apparent viscosity ($\mu_a = m.\dot{\gamma}^{n-1}$) near the wall. The flow thus moves more freely, and a greater uniformity in the distribution of the velocity is obtained (flattened shape). It should be noted that for *n*≈0.0, the shape of the velocity profile is almost flat with constant amplitude along *r*, this is a reminder to the case of gas characterized by very low viscosities. Thus, velocity is characterized by a mean value (flow/Section).

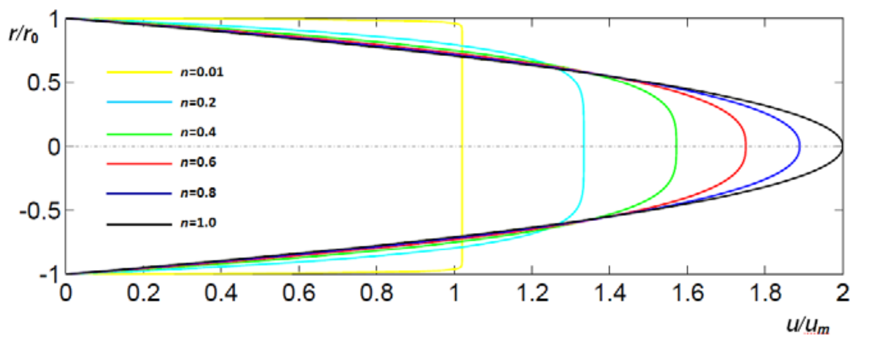

Fig.2 Effect of *n* on the velocity profile.

2.3 Energy equation

The dimensionless velocity profile expression is injected into the energy equation below, in which the last term on the right describes the viscous dissipation:

$$\rho C_p u\frac{\partial T}{\partial z} = k\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \tau_{rz}\frac{\partial u}{\partial r} \tag{3}$$

Where: k, ρ *and* C_p are thermal conductivity, density and specific heat respectively.

We replace u and τ_{rz} by their expressions, we obtain:

$$u_m\left(\frac{3n+1}{n+1}\right)\left[1-\left(\frac{r}{r_0}\right)^{\frac{n+1}{n}}\right]\left(\frac{\partial T}{\partial z}\right) = \frac{k}{\rho C_p}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{m}{\rho C_p}\left(\frac{u_m}{D}\right)^{n+1}\left(\frac{6n+2}{n}\right)^{n+1}\left(\frac{r}{r_0}\right)^{\frac{n+1}{n}} \tag{4}$$

This equation governs the radial and axial temperature changes.

To facilitate the numerical solution of the energy equation, the following dimensionless parameters are introduced:

$$R = \frac{r}{D}\ ;\ Z = \frac{z}{D}\ ;\ U = \frac{u}{u_m}\ ;\ \theta_{(T imp)} = \frac{T_w - T}{T_w - T_e} \tag{5}$$

Used in equation (4), we get the following dimensionless equation:

$$\left(\frac{3n+1}{n+1}\right)\left[1-\left(\frac{R}{0.5}\right)^{\frac{n+1}{n}}\right]\left(\frac{\partial\theta}{\partial Z}\right) = \frac{1}{Pe}\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta}{\partial R}\right) - \frac{Br}{Pe}\left(\frac{6n+2}{n}\right)^{n+1}\left(\frac{R}{0.5}\right)^{\frac{n+1}{n}} \tag{6}$$

Where:

$$Pe = \frac{u_m.\rho.C_p.D}{k} \quad \text{(Peclet number)} \tag{7}$$

$$Br = \frac{m u_m^{\,n+1}}{k D^{n-1}(T_w - T_e)} \quad \text{(Brinkman number)} \tag{8}$$

The last dimensionless number appears after nondimensionalization, is the source term due to viscous dissipation. This heating phenomenon by viscous friction will be more pronounced when the *Br* value is high and vice versa.

2.4 Boundaries conditions

$$\begin{aligned} &Z = 0 \quad \rightarrow \ \theta = 1.0 \\ &R = 0 \quad \rightarrow \partial\theta/\partial R\big|_{R=0.5} = 0 \\ &R = 0.5 \rightarrow \theta = 0.0 \end{aligned} \tag{9}$$

3. Numerical Resolution

The finite difference method is chosen to discretize the terms following *R* and *Z* directions. Simple implicit scheme is then followed for the rearrangement of discretized terms to make the use of the Thomas algorithm possible. A regular grid in *R* direction is selected while a gradually growing mesh following *Z* is adopted. For our case, and after the convergence study we took ΔR=10^{-3} (501 Nodes), whereas following *Z*, we took ΔZ_0=10^{-7} with an amplification factor of 1.005.

3.1 Code validation

In order to make the present work valuable, two validations were made. The first on the evolution of the Nusselt number along the pipe by neglecting the effect of viscous dissipation with *Pe*=1.0. Our results compared to the case of a Newtonian fluid with those of reference [1] for two heating types (Table.1). A very good agreement between the two works is observed.

A second validation is done for the value of Nu established for different values of *n*. Our results for specified *n* were compared with those of reference [4] for the case of temperature imposed, and those of reference [3] for an imposed flux case. Our results show a good accuracy with literature (Table.2).

Table 1 Evolution of *Nu* along *Z*. *n*=1.0; *Br*=0

Z	Imposed flux		Imposed Temperature	
	This work	Ref [1]	This work	Ref [1]
0,0001	27,30328	27.2760	22,30369	22,279
0,0005	15,82263	15.8130	12,83301	12,824
0,001	12,54433	12.5380	10,13574	10,13
0,005	7,498165	7.4937	6,00512	6,0015
0,01	6,15188	6.1481	4,91886	4,9161
0,02	5,20167	5.1984	4,17471	4,1724
0,05	4,51721	3.5139	3,71199	3,71
0,1	4,37786	3.3748	3,65981	3,6581
0,2	4,36912	3.3637	3,65842	3,6568

Table 2 Values of established *Nu* for different *n*.

n	Imposed Temperature		Imposed Flux	
	This work	Ref [4]	This work	Ref [3]
0,2	17,71237	17,2681	3,44943	3,4474
0,5	11,64363	11,5239	2,4559	2,44881
1	9,58713	9,5225	1,37522	1,3714
2	8,54769	8,5019	0,30672	0,3026
5	7,91391	7,8818	0,00152	0,0015

4. Results and discussion

View to the many parameters involved in the problem (*Br*, *n*, heating condition), the results will be presented for the case of heating then for the case of cooling.

Generally, the generation of heat due to viscous dissipation is characterized by a gradual increase of the temperature within the fluid. This growing increase with increasing *Br*, manages to bring the fluid temperature to levels sometimes exceeding the temperature of the wall. This means that fluid starts from an axial position to heat the wall. Details on this point with many results are given in reference [6, Chap 3]. Referral to this reference is the fact that its presentation requires many figures which the paper space constraint prevents.

An important element in the heat transfer problems is the bulk temperature θ_m. In addition it goes directly into the calculation of the Nusselt. So, it serves as an evaluation gauge for the overall evolution of the fluid temperature. This is the fact that it is a weighted average (by the velocity distribution), which gives an idea about the thermization process (or cooling).

In the following, we will present the axial evolution of the bulk temperature, from the inlet to the fully developed stage. Two values of *n* are chosen (*n*=0.1 and *n*=1.0), for five values of *Br*. This is done to the case of a heating and that of cooling. We note for the case of cooling, *Br* is simply taken negative (see its expression). In addition, the Peclet number is taken equal 100.0, to neglect the axial conduction.

4.1. Evolution of the bulk temperature

4.1.1. Heating at the wall

The case of heating at the wall is shown in Figure (3). The dotted line is for θ_w = 0.0. It is clear that the dimensionless temperature becomes increasingly negative when *Br* increases, indicating a strong heating, particularly in the central area due to the generation of heat by viscous friction. The rheological index *n* also affects this evolution, where high heat generation is observed for *n*=1.0 compared to *n*=0.1. This is explained by the fact that the decrease of *n* results in a decrease on the viscosity, and therefore the viscous friction directly related.

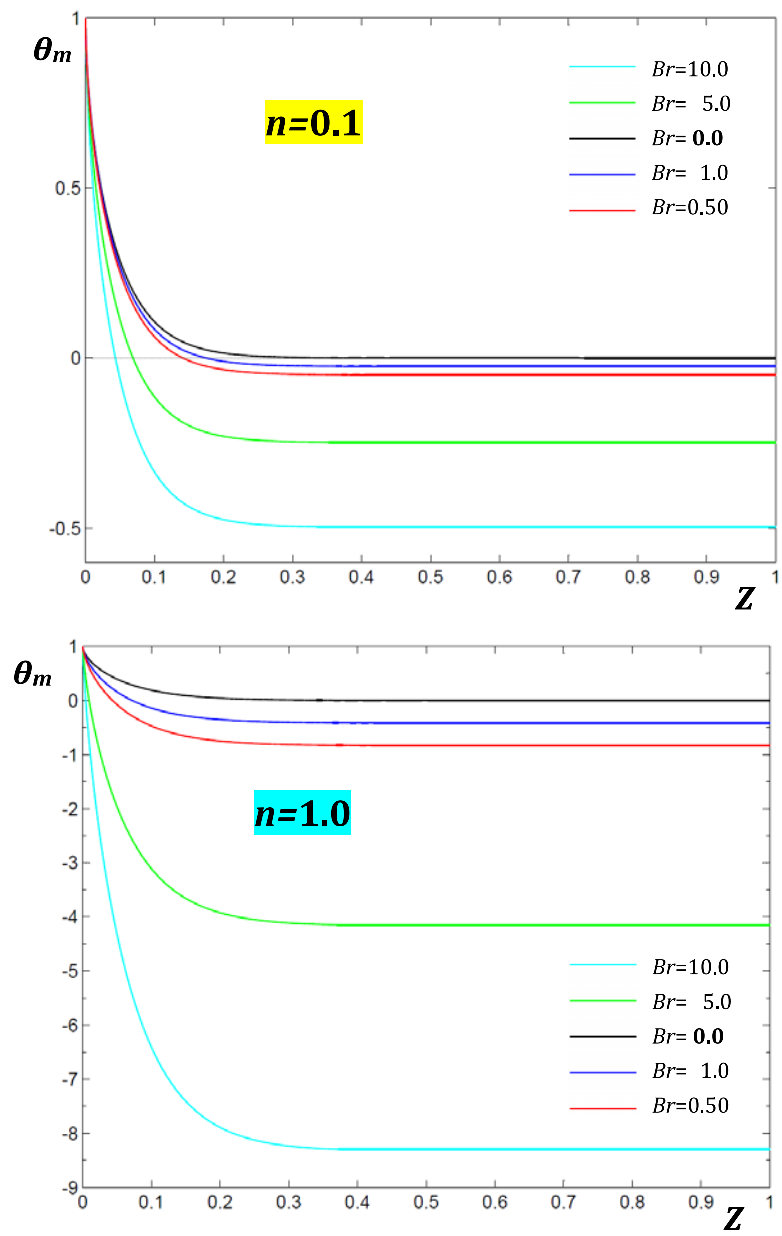

Fig.3 Evolution of θ_m along the pipe for different *n* and *Br*. *Heating case.*

From the two sub-figures, we can see approximately, the axial position from which the transmitted heat flux changes direction. This position becomes closer to the inlet when *Br* and /or *n* increase.

4.1.2. Cooling at the wall

Contrary to the previous case, the bulk temperature in this case increases with the increase of *Br*. The values of the bulk temperature for *n*=1.0 are larger than those corresponding to *n*=0.1 (Figure 4).

A new phenomenon is observed for *Br*= -5.0 and *Br* = -10.0 for *n*=1.0. The bulk temperature changes slope from the entrance, where very high values are recorded reaching 8.0, while for other cases it remains below 1.0. This -in our opinion- is explained by the fact that the great value of *Br* associated with high *n*, lead to strong heating by viscous friction. This friction intensively grows the dimensionless temperature even before the cooling process begins (i.e. just at the entrance).

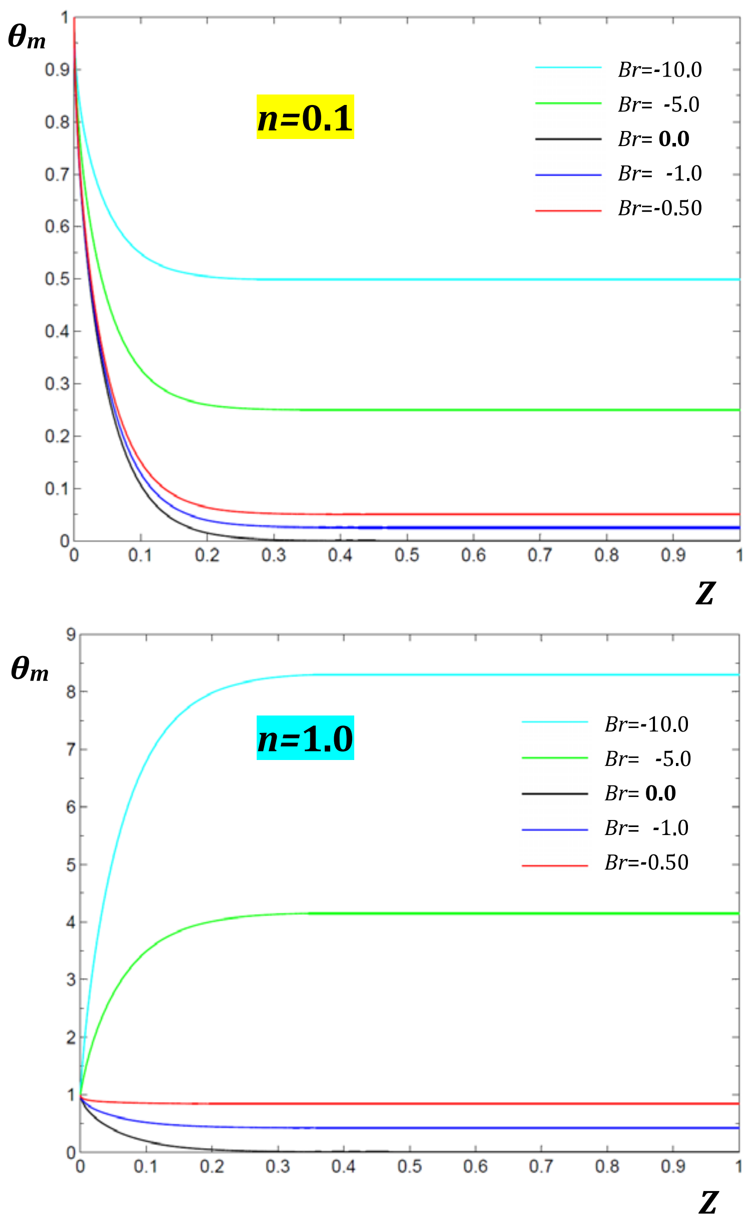

Fig.4 Evolution of θ_m along the pipe for different *n* and *Br*. *Cooling case.*

This phenomenon is not observed for *n*=0.1, the fact that the shear-thinning reduces the friction between the fluid layers. It is clear that there is a value of *n* from which we begin to see this phenomenon.

4.2. Evolution of the Nusselt number

Nusselt number, which characterizes the intensity of the convective heat exchange, is another important parameter. The effects of the rheological index *n* and the Brinkman number *Br* are presented in figure (5) for heating case, and in figure (6) for the cooling one.

We note that we are limited to *n*=0.1 and *n*=1.0, for lack of space, other values are plotted in [6, Chap 3].

4.2.1. Heating at the wall

In this part the Nusselt evolution was presented for *n*=0.1 and *n*=1.0 and five *Br* values (Figure 5). The black line is for the case *Br*=0.0 (negligible viscous dissipation), where the habitual curves are found and the value of *Nu* in the establishment is mentioned. But when *Br* is not zero, we see an evolution that seems strange for the two sub-figures. Indicating that the rheological index *n* and *Br*, do not affect the evolution shape but affect the *Nu* value.

The Nusselt decreases gradually from the inlet. But it continues to decrease to very low negative values, and then at an axial position, it abruptly rises where a jump is recorded. Finally, it goes down to its establishment value.

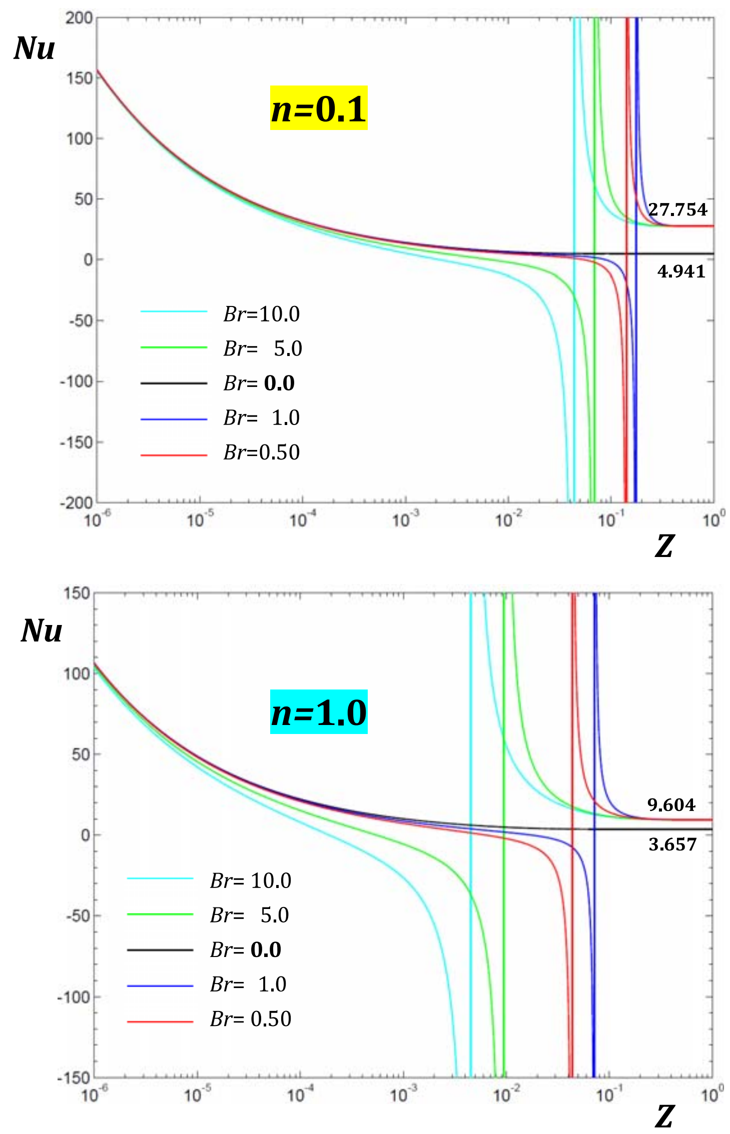

Fig.5 Evolution of *Nu* along the pipe for different *n* and *Br*. *Heating case.*

The explanation for this change is as follows: at the entrance, the fluid temperature and that of bulk temperature are very close to the inlet temperature (θ_e). This will give a negative term in the numerator close to 1.0 and a negative term close to 1.0 multiplied by ΔR in the denominator ($Nu = +\frac{1}{\theta_m}\partial\theta/\partial R\big|_{R=0.5}$). This will give a high positive *Nu*. Away from the inlet, the fluid becomes hot, and under the effect of the viscous dissipation its temperature exceeds that of the wall, negative values are obtained, recalling that θ_m is always lower than that of the wall. After some distance, θ_m becomes equal to that of the wall (=0.0), and the jump in the value of *Nu* is observed, its value becomes positive and a decrease until the establishment was observed thereafter. When increasing, *Br* moves the point of singularity to the inlet since it intensifies the heat generation by viscous dissipation, which accelerates the phenomena described above. The rheological index engenders the opposite effect.

The analysis of the sub-figures shows that Nusselt curves met at the same value at establishment, regardless of the value of *Br*. This is explained by the fact that its calculation is conditioned by θ_W fixed along the whole pipe length.

4.2.2. Cooling at the wall

The case of cooling is now considered. The Nusselt numbers evolutions are plotted in Figure (6) for the same values of *n* considered in the previous case, and five negative *Br*. It is clearly seen that *Nu* curves for *n*=0.1 have minimums, followed by lifts until establishment. This is valid for all *Br* supposed. It is explained by the fact that θ_{N-1} (close to wall) decreases faster than θ_m near the entrance. This process continues until reaching the *Nu* minimum. With the generation of heat by viscous dissipation θ_m also rapidly decreases and a lift in *Nu* shape is obtained.

The rheological index *n* has a remarkable effect on the evolution of *Nu*, as its increase favors the viscous dissipation effect as detailed above. This can be observed for the curves of *Br*=-0.5 and *Br*=-1.0, where a similar behavior to that observed for *n*=0.1 is obtained. Contrary for the other *Br* and in particular for strong *n*, the Nusselt no longer presents the minimum, as consequence of the great temperature field disturbance due to intense viscous dissipation that increases rapidly.

The same phenomenon at the establishment stage is reproduced for the case of cooling, where the curves at different *Br* are joined at the same value.

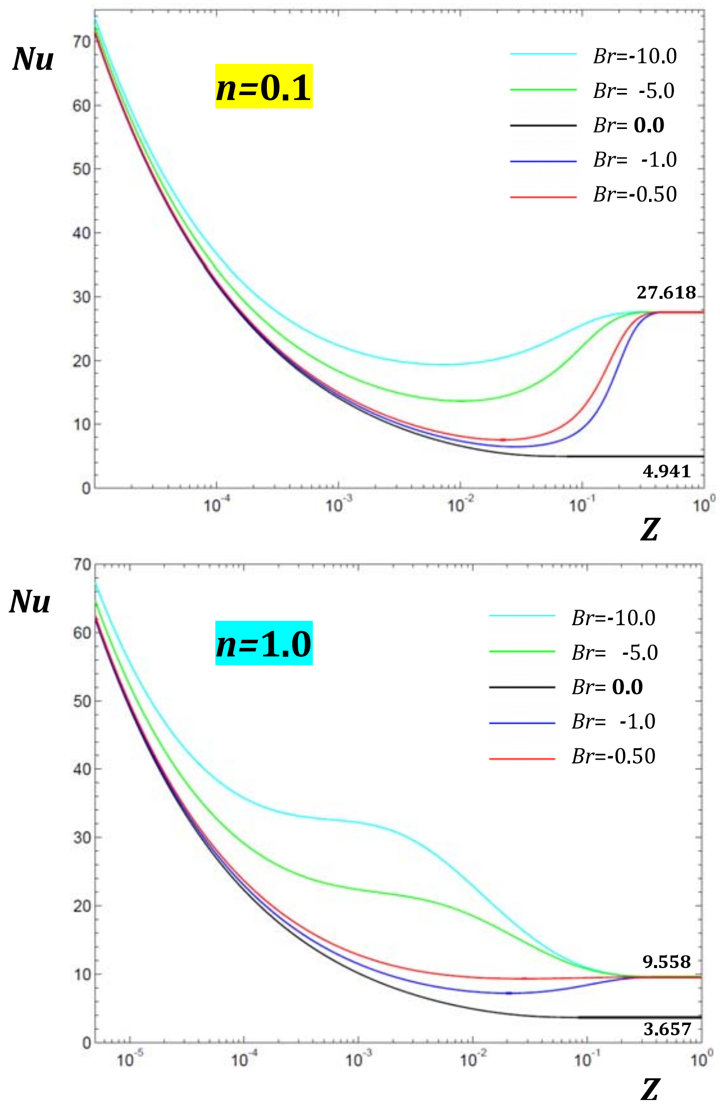

Fig. 6 Evolution of *Nu* along the pipe for different *n* and *Br*. *Cooling case.*

5. Conclusions

The results obtained from this Numerical work show that:

- The viscous dissipation increases with *Br*, and increasingly high temperatures are recorded;
- The decrease in rheological index *n* reduces the effect of viscous dissipation due to the reduction of friction between the fluid layers;
- A singularity in the Nusselt evolution is observed in the case of heating. This singularity becomes closer to the entrance when *Br* and/or *n* increase;
- In the case of cooling, the Nusselt number increases near the entrance with the increase of *Br*. Its evolution is characterized with a slope down followed by a lift when viscous dissipation becomes intense. This result is also affected by *Br* an *n* values;
- For the two cases supposed, the fully developed *Nu* value is independent to the change of *Br*. But it is greater of that for *Br*=0.0.

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