International Journal of Mechanics and Energy (IJME)
Vol. 4, Issue 1, 2016, ISSN: 2286-5845

Effects of inclination angle on natural convection in enclosures filled with a shear thinning Thermo-dependent fluids

Mourad Kaddiri[1]*,Mohamed Lamsaadi[1],HassanElharfi[1], Mohamed Naimi[1] , Abdelghani Raji[1] , Mohammed Hassnaoui[2]

[1]*Sultan MoulaySlimane University, Faculty of Sciences and Technologies, Laboratory of flows and Transfers Modeling (LAMET), B.P. 523, Beni-Mellal, Morocco*

[2]*Cadi Ayyad University, Faculty of Sciences Semlalia, Laboratory of Fluid Mechanics and Energetics (LMFE), B.P. 2390, Marrakech, Morocco*

Abstract: Natural convection heat transfer in inclined square cavity filled with thermo-dependent power-law fluids has been investigated numerically. The enclosure considered here is heated and cooled with uniform fluxes from the horizontal walls, while the verticals ones are adiabatic. The effects of the governing parameters, which are the thermo-dependence number $(0 \le m \le 10)$, the flow behavior index $(n = 0.6)$, the Rayleigh number $(Ra = 10^4)$ and the angle of inclination $(0° \le \phi \le 120°)$, on flow structure and heat transfer characteristics have been examined. Results are presented in the form of streamline and isotherm plots as well as the variation of the Nusselt number under different conditions.

Keywords: Heattransfer; Natural convection; Non-Newtonianfluids; Tilted square cavity; Thermo-dependentBehavior.

Nomenclature

b: temperature coefficient.
g: acceleration due to gravity (m/s^2)
H': height or width of the enclosure (m)
k: consistency index for a power-law fluid at the reference temperature $(Pa \cdot s^n)$
m: thermo-dependence number
n: flow behavior index for a power-law fluid.
Nuv: the vertical average Nusselt number.
Pr: generalisedPrandtl number.
q': constant density of heat flux (W/m^2)
Ra: generalised Rayleigh number.
T: dimensionless temperature, $(= (T' - T'_r)/\Delta T^*)$
T'_r: reference temperature (K)
ΔT^* : characteristic temperature $(= q'H'/\lambda)$ (K)
(u,v) dimensionless horizontal and vertical velocities $(= (u',v')/(\alpha/H'))$
(x,y) : dimensionless horizontal and vertical coordinates $(= (x',y')/H')$

Greek symbols

α : thermal diffusivity of fluid at the reference temperature (m^2/s)
β: thermal expansion coefficient of fluid at the reference temperature $(1/K)$
λ: thermal conductivity of fluid at the reference temperature $(W/m \cdot °C)$
μ: dynamic viscosity for a Newtonian fluid at the reference temperature $(Pa \cdot s)$
μ_a: dimensionless effective viscosity of fluid.
ρ: density of fluid at the reference temperature (kg/m^3)
Ω: dimensionlessvorticity, $(= \Omega'/(\alpha/H'^2))$
ψ: dimensionless stream function, $(= \psi'/\alpha)$

Superscript

':dimensional variables

Subscripts

a: effective variable
max: maximum value
r: reference value taken at the cavity centre

* **Correspondingauthor:**MouradKaddiri
E-mail: mouradkadiri@usms.ma

1. Introduction

Natural convection from density variations within a non-isothermal fluid under the gravity effect has received considerable attention in the literature. Useful review can be found in the article and book by Ostrach [1] and Gebhart et al. [2], respectively, where most of the fluids considered are of Newtonian behaviour. However, most of materials that are of interest in a variety of manufacturing processes, exhibit non-Newtonian behaviours, which implies that, the shear stress is not proportional to the shear rate [2].

These type of fluids have received, through the decades, considerable attention by many researchers to investigate it in many geometrical configurations and under various boundary conditions. In this frame, the first numerical study concerning natural convection of non-Newtonian fluids confined in a differentially heated enclosure seems to be due to Ozoe and Churchill [3]. These authors have identified the critical conditions related to the onset of convection and have shown that the critical Rayleigh number and the average Nusselt increase and decrease, respectively, with the index of behaviour. However, their results underestimate those obtained experimentally by Tien et al. (1969) [4].

After nearly two decades, Turki [5] investigated numerically a problem of natural convection in a closed rectangular cavity, differentially heated and filled with non-Newtonian fluids. His results were found to be in more or less satisfactory agreement with those obtained experimentally, one year before, by Cardon [6]. After that, Lamsaadi et al. [7] have studied natural-convection heat transfer in a horizontal enclosure containing non-Newtonian power-law fluids and heated from the bottom by a constant heat flux. The flow patterns, temperature distribution, and heat transfer rate are found to be rather sensitive to the non-Newtonian power-law behaviour but not to the large Prandtl number values (Pr>100). More recently, Allaoui et al. [8] analysed the onset of convection of power-law fluids in a shallow cavity heated from below by a constant heat flux. It was noted that, for shear-thinning fluids, the onset of convection is subcritical; whereas, for shear thickening fluids, convection is found to occur at a supercritical Rayleigh number equal to zero.

The case of an inclined enclosure, was also investigated analytically and numerically by Lamsaadi et al. [9,10]. Heat transfer of dilatant power-law fluids in two dimensional tilted enclosures heated from below has been investigated numerically by Vinogradov et al. [11]. It was reported that, despite significant variation in heat transfer rate both Newtonian and non-Newtonian fluids exhibit similar behavior with the transition from multi-cells flow structure to a single-cell regime. Recently, Khezzar et al. [12].extended the work of Kim et al.[13]. if the behaviour index is greater than unity, while examining the effect on the rheological behaviour of the heat transfer rates for different angles of inclination of the cavity. The authors observed, depending on the Rayleigh number, the existence of a critical value of the inclination angle for which the heat transfer rate is maximum.

On the other hand, most of the reported studies on natural convection involving non-Newtonian fluids ignored the dependence of the effective viscosity on temperature (thermo-dependence in other words); which constitutes another challenging problem to deal with. This can be a serious assumption, since in many cases this dependence has a significant influence on flow and heat transfer as proved, earlier, experimentally by Scirocco et al. [14] and numerically by Shin and Cho [15] whose results, of local Nusselt numbers for a polyacrylamide (Separan AP-273) solution, show 70-300% heat transfer enhancement over those of a constant-property fluid.

For natural convection phenomenon in such media, the literature review does not show an important number of investigations carried out in this area, especially for simple geometries such as square and rectangular cavities. Among the few studies conducted in this context, we can cite that of Turki [5], who found that, for power-law fluids filling a rectangular cavity differentially heated from the vertical sides, the consistency thermo-dependence affects substantially the flow structure and the local heat transfer but not significantly the overall one. Lately, Solomatov and Barr [16,17] examined numerically such an effect, for the same types of fluids as those considered by Turki [5], on the onset of the Rayleigh-Bénard convection and found that a decrease of the viscosity with the temperature anticipates the convection onset. Recently, Kaddiri et al. [18] analysed the effects of temperature dependence of consistency K on natural convection of power-law fluids in square enclosures with differentially-heated horizontal walls subjected to constant wall heat fluxes.It emerges from such a study, the viscosity variations with temperature act to reduce the convective zone thickness, giving rise to a conductive lid regime that reduces notably the heat transfer.

Therefore, to contribute to a better understanding of the thermo-dependence effects on thermal buoyancy convection in such media, a numerical study is performed to investigate the temperature-dependent viscosity effect on natural convection flow and heat transfer in a tilted square cavity confining non-Newtonian power-law fluids. The enclosure considered here is heated and cooled with uniform fluxes from the horizontal walls, while the verticals ones are adiabatic.

2. Mathematical Formulation

2.1. Problem Statement and Viscosity Model

Plotting of considered model is shown in Figure 1.with coordinates. It consists of a two-dimensional square enclosure of size $H' \times H'$ subjected to vertical uniform densities of heat flux, q' when inclination angle is zero. Remaining walls are adiabatic. The enclosure is filled with non- Newtonian fluid, which is incompressible and laminar.

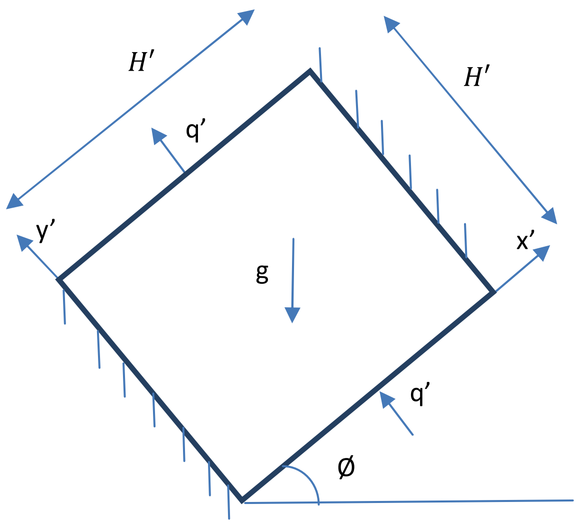

Figure 1. Sketch of the geometry and coordinates system

The non-Newtonian fluids considered here are those whose rheological behaviorscan be approached by the power-law model, due to Ostwald-de Waele, which, in terms of laminar effective viscosity, can be written as follows:

$$\mu'_a = k_T \left[2\left(\left(\frac{\partial u'}{\partial x'}\right)^2 + \left(\frac{\partial v'}{\partial y'}\right)^2 \right) + \left(\frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'}\right)^2 \right]^{\frac{(n_T-1)}{2}} \quad (1)$$

The two empirical parameters $n_T$and k_T appearing in (1), are the flow behavior and consistency indices, respectively. They are, in general, functions of the temperature, but in most of cases the temperature-dependence of n_T can be ignored $(n_T = n)$ since it is weak compared to that of k_T [14,19], which is described by the Frank-Kamenetski exponential law[20]:

$$k_T = ke^{-b(T'-T_r')} \quad (2)$$

reflecting the viscosity diminution with the temperature, where b is an exponent related to the flow energy activation and the universal gas constant, and T'_r is a reference temperature.

Note that for $n = 1$ the behavior is Newtonian and the consistency is just the viscosity. For $0 < n < 1$, the effective viscosity decreases with the amount of deformation and the behavior is shear-thinning. Conversely, for $n > 1$, the viscosity increases with the amount of shearing, which implies that, the fluid behavior is shear-thickening.

2.2. Governing Equations and Boundary Conditions

On the basis of the assumptions commonly adopted in natural convection problems, the dimensionless governing equations for Boussinesq-temperature-dependent viscosity fluids, written in terms of vorticity, Ω, temperature, T, and stream function, ψ, are as follows:

where

$$\frac{\partial \Omega}{\partial t} + \frac{\partial (u\Omega)}{\partial x} + \frac{\partial (v\Omega)}{\partial y} = P_r\left[\mu_a \nabla^2 \Omega + 2\vec{\nabla}\mu_a . \vec{\nabla}\Omega\right] + S_\Omega , \quad (3)$$

$$\frac{\partial T}{\partial t} + \frac{\partial (uT)}{\partial x} + \frac{\partial (vT)}{\partial y} = \nabla^2 T, \quad (4)$$

and

$$\nabla^2 \psi = -\Omega, \quad (5)$$

where

$$u = \frac{\partial \psi}{\partial x} , \; v = -\frac{\partial \psi}{\partial y} , \; \Omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} ,$$

$$S_\Omega = P_r \left[\left[\frac{\partial^2 \mu_a}{\partial x^2} - \frac{\partial^2 \mu_a}{\partial y^2}\right] \left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right] - 2\frac{\partial^2 \mu_a}{\partial x \partial y}\left[\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right] \right] + P_r R_a \left(\frac{\partial T}{\partial x}\cos(\emptyset) - \frac{\partial T}{\partial y}\sin(\emptyset)\right)$$

and

$$\mu_a = e^{-mT} \left[2\left(\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 \right) + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \right]^{\frac{(n-1)}{2}}$$

For the present problem, the appropriate non-dimensional boundary conditions are:

$$u = v = \psi = \frac{\partial T}{\partial x} = 0 \text{ for} x = 0 \text{ and } x = 1 \quad (6)$$

$$u = v = \psi = \frac{\partial T}{\partial y} + 1 = 0 \text{ for} y = 0 \text{ and } y = 1 \quad (7)$$

In addition to the flow behavior index, n, and the inclination angles, Ø, three other dimensionless parameters appear in the above equations, namely, the Pearson, generalized Prandtl and Rayleigh numbers defined, respectively, as:

$$m = -\frac{1}{k_T}\frac{dk_T}{dT} = -\frac{dLn(k_T/k)}{dT}, P_r = \frac{(k/\rho)H'^{2-2n}}{\alpha^{2-n}} \text{ and}$$
$$R_a = \frac{g\beta H'^{2n+2} q'}{(k/\rho)\alpha^n \lambda} \quad (8)$$

The Pearson number (8), which is a new dimensionless quantity taking place in this study, measures the effect of temperature change on the effective viscosity.

2.3. Heat Transfer

The steady solution has been used to calculate the average Nusselt number in the horizontal and vertical directions, respectively, defined as:

$$Nu_v = \frac{q' H'}{\lambda \Delta T'} = \frac{1}{\Delta T} = \frac{1}{T(x,0) - T(x,1)} \quad (9)$$

2.4. Heatlines Formulation

The visualization of the paths followed by the heat flow through the enclosure requires the use of the heatlines concept, which consists of lines of constant heat function, H, that are defined, according to Kimura and Bejan [21], from the following equations

$$\frac{\partial H}{\partial y} = uT - \frac{\partial T}{\partial x}, \; -\frac{\partial H}{\partial x} = vT - \frac{\partial T}{\partial y} \quad (10)$$

whose derivation, with respect to x and y, and combination give rise to

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = -\frac{\partial vT}{\partial x} + \frac{\partial uT}{\partial y} \quad (11)$$

To obtain the boundary conditions associated with (13), an integration of (10), along the four cavity walls, is necessary, which gives:

$$H(0, y) = H(0,0) \text{for} x = 0 \quad (12)$$

$$H(x, 1) = H(0,1) - x \text{for} y = 1 \quad (13)$$

$$H(1, y) = H(1,1) \text{for} x = 1 \quad (14)$$

$$H(x, 0) = H(1,0) + 1 - x \text{for} y = 0 \quad (15)$$

Finally, the solution of (11) yields the values of H, in the computational domain, whose contour plots provide the heatline patterns. Note that only the differences between the values of H are required instead of its intrinsic ones, which offers the possibility to choose $H(0,0) = 0$ as an arbitrary reference value for H.

3. Solution Procedure

The two-dimensional governing equations have been discretized using the second order central finite difference methodology with a regular mesh size. The integration of (3) and (4) has been performed with the Alternating Direction Implicit method (ADI), originally used for Newtonian fluids and successfully experimented for non-Newtonian power-law fluids [3,5,7].To satisfy the mass conservation, (5) has been solved by a Point Successive Over Relaxation method (PSOR) with an optimum relaxation factor calculated by the Frankel formula [22]. A grid of 81×81 has been required for obtaining adequate results.

The numerical results from the code have been validated using the benchmark data of de Vahl Davis [23], Turki [5] and Ouertatani [24] for natural convection of Newtonian and non-Newtonian fluids in square enclosures with differentially heated vertical walls and an excellent agreement was obtained (see Table 2 of Ref. [18] and Ref. [25]).

4. Results and Discussion

As was reported in the past by Ozoe and Churchill [3], Lamsaadi et al. [7] and many others Turan et al.[26], the convection is rather insensitive to Pr variations, provided that this parameter is large enough as it is the case for the non-Newtonian fluids and for a large category of fluids having a Newtonian behavior. Therefore, Pr is not considered as an influencing parameter in this study and the simulations are conducted with $P_r \to \infty$.

To examine the inclination angles and the thermo-dependency effects, Calculations were made forvarious values of Pearson number (0<m<10), inclination angles (0° <Ø<150°), while the flow behavior index andRayleigh numbers are fixed as n=0.6 and Ra= 10^4.

Figure2.illustrates the streamlines (left), isotherms (middle) and heatlines (right) at different inclination angles for Ra = 10^4 and different value of n=0.6. Again, this figure makes a comparison between the constant and temperature-dependent viscosity cases of fluid. It is

observed that the shape of the main cell is sensitive to the inclination angle and thermo-dependency of fluid.In addition, a close inspection of the isotherms shows that the latter change moderately by increasing Ø since their important distortion, observed for the case (Ø = 0°), tends to vanish for the case (Ø= 120°).

For, the heatlines, an increase of the inclination angle reduces the number of internal recirculation, which shows a decrease in intensity of convection.Moreover, the effect of thermo-dependence remains the same for all the cases studied, i.e. it destabilizes the flow and always form two regions: a stagnant and the other active, whatever the value of Ø

Concerning heat transfer, Figure3.illustrates the variation of average Nusselt number at different inclination angle for m=0. and m=10. As seen from this figure, the average Nusselt number increases for the tilt angle values from Ø= 0° to Ø= 60° reaching a maximum and then decreases. In addition, figure3shows that the effect of inclination angles does not change the effect of thermo-dependence of the viscosity on the heat transfer.

5. Conclusion

In this paper, numerical calculations have been presented for the natural convection flow in an inclined square cavity, filled with power-law fluids and submitted to vertical uniform heat fluxes, while the vertical walls are adiabatic. The study is focused particularly on temperature-dependent viscosity effect on natural convection flow and heat transfer in a tilted square cavity at a wide variety of angles of inclination.

It emerges that the thermo-dependent behaviourdestabilizes the flow and always form two regions: a stagnant and the other active. This effect manifests its self only when convection is weak.

Depending on the Rayleigh number, it is expected the existence of a critical value of the inclination angle for which the heat transfer rate is maximum.In addition, the effect of inclination angles does not change the effect of thermo-dependence of the viscosity on the heat transfer.

References

[1] S. Ostrach, “Natural convection in enclosures,” J. Heat Transfer. 1988. 110. P. 1175–1190.

[2] B. Gebhart, Y. Jaluria, R. L. Mahajan, and B. Sammakia, “Buoyancy-induced flows and transport, ” chap. 16, Hemisphere, Washington, DC, 1988.

[3] H. Ozoe, and S.W. Churchill, “Hydrodynamic stability and natural convection in Ostwald–De Waele and Ellis fluids: the development of a numerical solution, ”AIChE J., 18, pp. 1196-1207, 1972.

[4] C. Tien, H. S. Tsuei, and Z. S. Sun, Thermal Instability of a Horizontal Layer of Non-Newtonian Fluid Heated from Below, Int. J. Heat Mass Transfer, Shorter Commun.,vol. 12, pp. 1173–1178, 1969.

[5] S. Turki, Contribution to Numerical Study of Natural and Mixed Convection Heat Transfers in Confined Non-Newtonian Fluids, Ph.D. Thesis, CNAM, Paris, France, 1990.

[6] X. Cardon, Experimental Study of Natural Convection Within a Rectangular Cavity, in a Ostwald-de-Waele Fluids of Thermo-Dependent Proprieties, Doctorate thesis, Nantes University, France, 1989.

[7] M. Lamsaadi, M. Naimi, M. Hasnaoui, Natural convection of non-Newtonian power law fluids in a shallow horizontal rectangular cavity uniformly heated from below, Heat Mass Transfer 41 (2005) 239 –249.

[8] Z. Alloui, N. Ben Khelifa, H. Beji, P. Vasseur, A. Guizani, The onset of convection of power-law fluids in a shallow cavity heated from below by a constant heat flux, J. Non-Newt. Fluid Mech. 196 (2013) 70–82.

[9] M. Lamsaadi, M. Naimi, M. Hasnaoui, Natural convection in a tilted rectangular slot containing non-Newtonian power-law fluids and subject to a longitudinal thermal gradient, Numer. Heat Transfer, Part A 50 (2006) 561–583.

[10] M. Lamsaadi, M. Naimi, A. Bahlaoui, A. Raji, M. Hasnaoui, Multiple steady-state solutions for natural convection in a tilted rectangular slot containing non-Newtonian power-law fluids and subject to a transverse thermal gradient, Numer. Heat Transfer, Part A 51 (2007) 393–414.

[11] I. Vinogradov, L. Khezzar, D. Siginer, Heat transfer of non-Newtonian dilatant power-law fluids in square and rectangular cavities, J. Appl. Fluid Mech. 4 (2) (2011) 37–42.

[12] L. Khezzar, D. Siginer, I. Vinogradov, Natural convection of power law fluids in inclined cavities, Int. J. Therm. Sci. 53 (2012) 8–17.

[13] G. Bin Kim, J. Min Hyun, H. Sang Kwak, Transient buoyant convection of a power-law non-Newtonian fluid in an enclosure, Int. J. Heat Mass Transfer 46 (2003) 3605–3617.

[14] V. Scirocco, R. Devienne, and M. Lebouché, Ecoulement laminaire et transfert de chaleur pour un fluide pseudo-plastique dans la zone d'entrée d'untube, Int. J. Heat Mass Transfer, 28(1), pp. 91-99, 1985.

[15] S. Shin, Y.I. Cho, Laminar heat transfer in a rectangular duct with a non-Newtonian fluid with temperature

dependent viscosity, International Journal of Heat and Mass Transfer, vol. 37, no. 1, pp. 19-30, 1994.

[16] V.S. Solomatov, and A.C. Barr, Onset of convection in fluids with strongly temperature-dependent power-law viscosity, Physics of the Earth and Planetary Interiors, 155, pp.140-145, 2006.

[17] V.S. Solomatov and A.C. Barr, Onset of convection in fluids with strongly temperature dependent power-law viscosity: 2. Dependence on the initial perturbation, Physics of the Earth and Planetary Interiors, 2007, 165, pp. 1-13.

[18] M. Kaddiri, M. Naïmi, A. Raji and M. Hasnaoui, Rayleigh-Bénard convection of non-Newtonian power-law fluids with temperature-dependent viscosity, ISRN Thermodynamics, Vol. 2012, Article ID 614712, 10 pages, 2012, doi.10.5402/2012/614712.

[19] C. Nouar, Thermal convection for a thermodependent yield stress fluid in an axisymmetric horizontal duct, Int. J. Heat Mass Transfer, 2005, 48, pp. 5520-5535.

[20] N. J. Balmforth and A. Provenzale, Geophysical Aspects of Non-Newtonian Fluid Mechanics, vol. 582 of Liberal National Party, Springer, 2001.

[21] S. Kimura and A. Bejan, “The Heatline Visualization of Convective Heat Transfer, ” ASME Journal of Heat Transfer, volume 105, pages 916–919, 1983.

[22] P.J. Roache, Computational Fluid Dynamics, New Mexico: Hermosa Publishers, 1982.

[23] De Vahl Davis, G. “Natural convection of air in a square cavity: a bench mark numerical solution, ” Int. J. Num. Method Fluids, 1983, 3, pp. 249-264.

[24] N. Ouertatani, N. Ben Cheikh, B. Ben Beya, and T. Lili, “Numerical simulation of two-dimensional Rayleigh-Benard convection in an enclosure,” ComptesRendus, vol. 336, no. 5,pp. 464–470, 2008.

[25] M. Kaddiri, M. Naïmi, A. Raji and M. Hasnaoui, “Thermal convection within a square cavity filled with non-Newtonian power-law fluids and differentially heated with uniform heat fluxes, ” International Scientific Journal for Alternative Energy and Ecology, Vol. 88, N° 8, pp. 57-69, 2010.

[26] O. Turan, J. Lai, R.J. Poole, N. Chakraborty, Laminar natural convection of power law fluids in a square enclosure submitted from below to a uniform heat flux density, J. Non-Newt. Fluid Mech. 199 (2013) 80–95

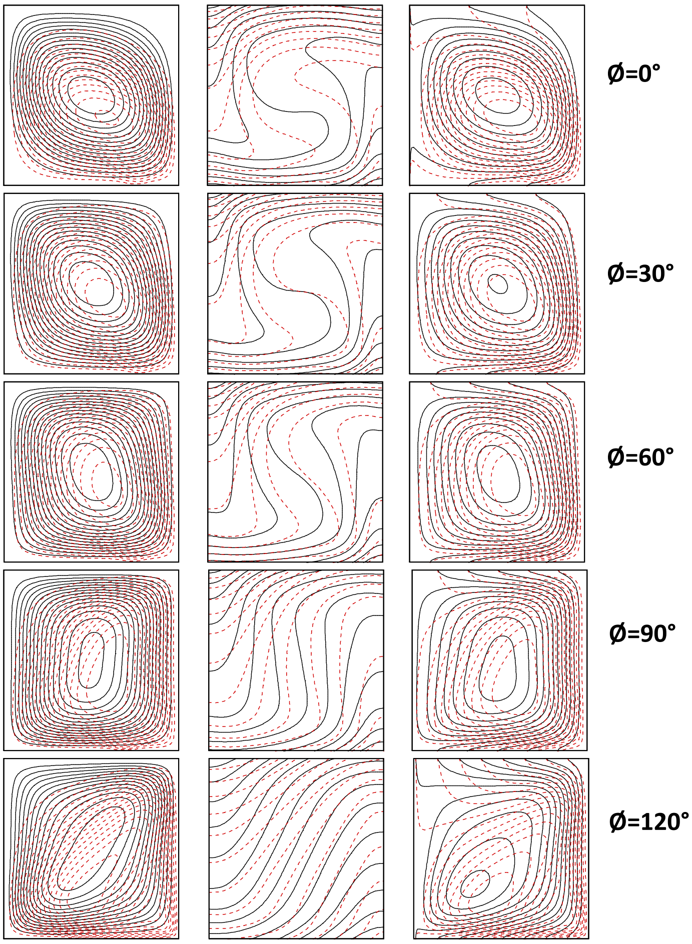

Figure 2. Streamlines (left), isotherms (medium) and heatlines (right) for Ra=10^4; n=0.6; m=0 (black solid line), m=10 (red dashdot line) and various value of Ø.

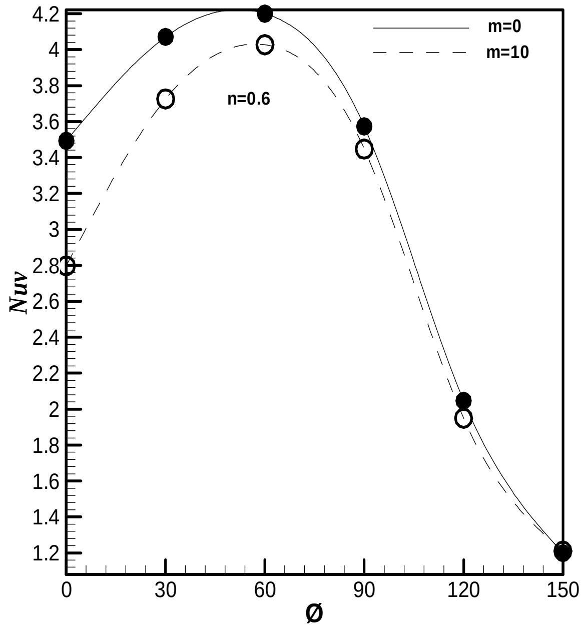

Figure 3. Evolution of vertical average Nusselt number in function of Ø for Ra=104; n=0.6; m=0 (black solid line) and m=10 (red dashdot line).